

The Fermi theory of beta-decay

Nuclear and Particle Physics

4th lecture

The beta-decay

atom

0,1 nm

nucleus

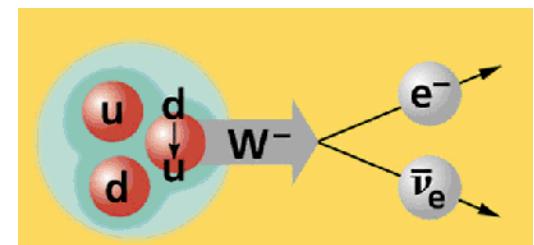
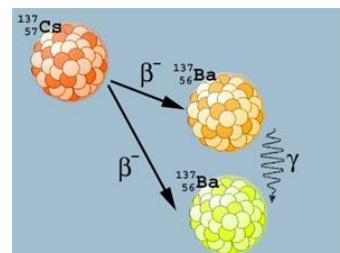
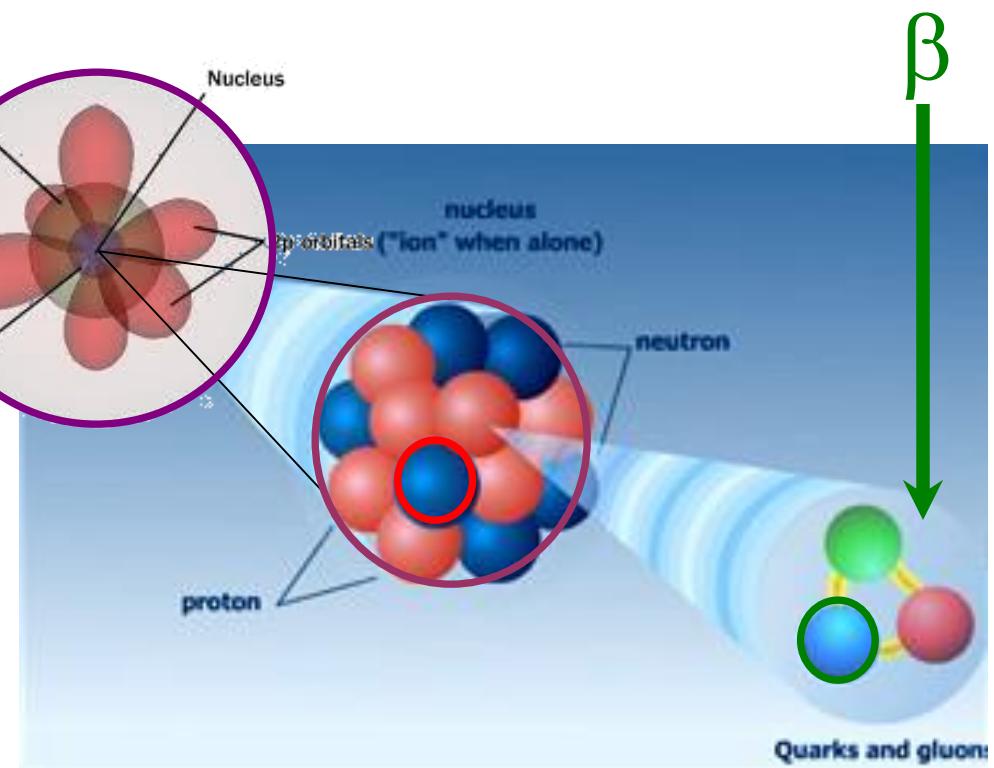
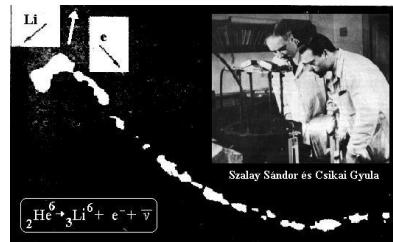
10 fm

nucleon

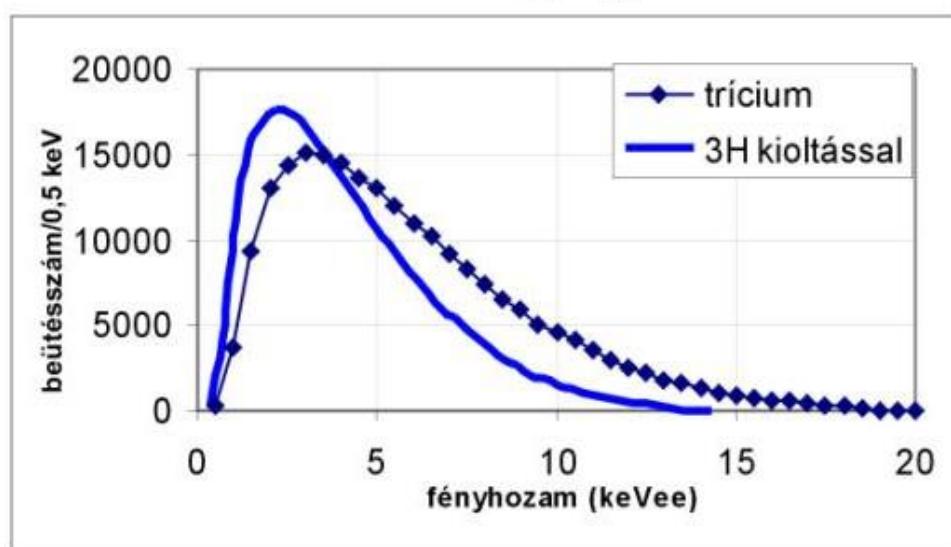
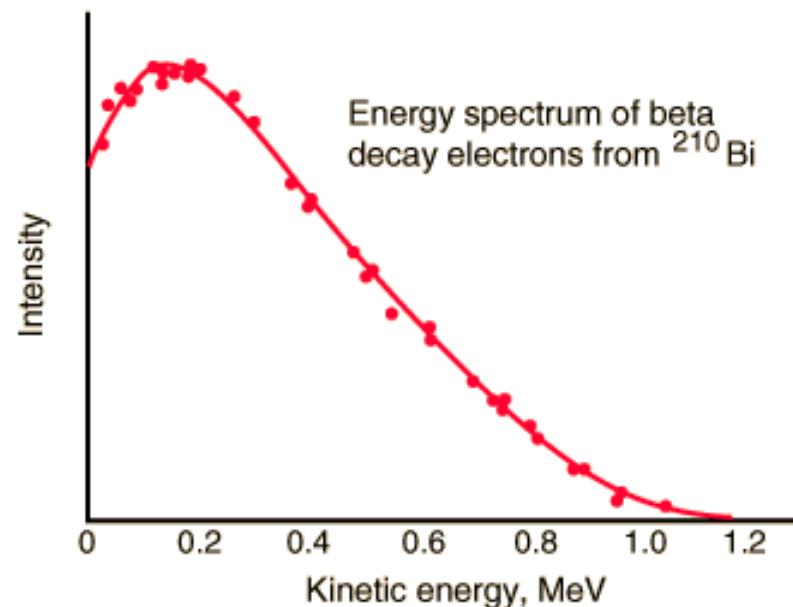
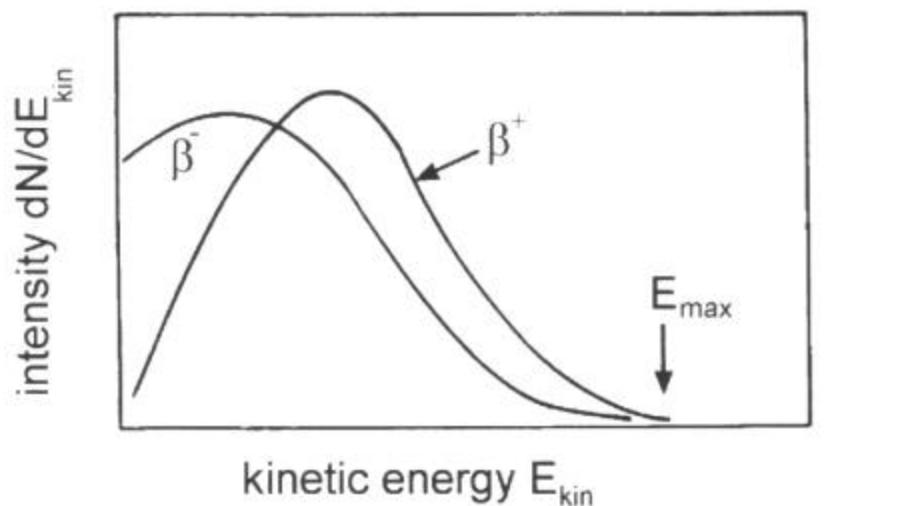
1 fm

quark
„elemi”

„elemi”

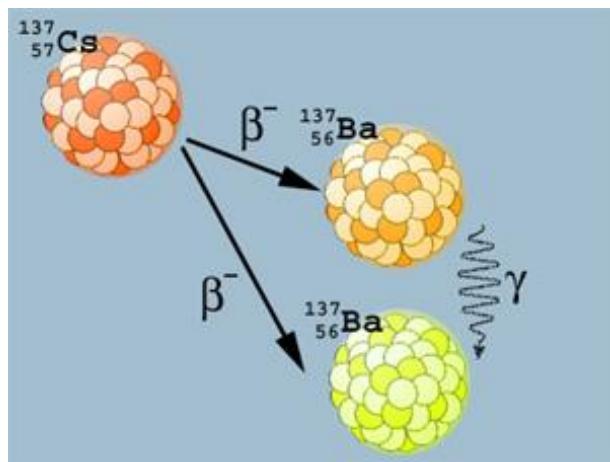


Energy spectra in beta-decay

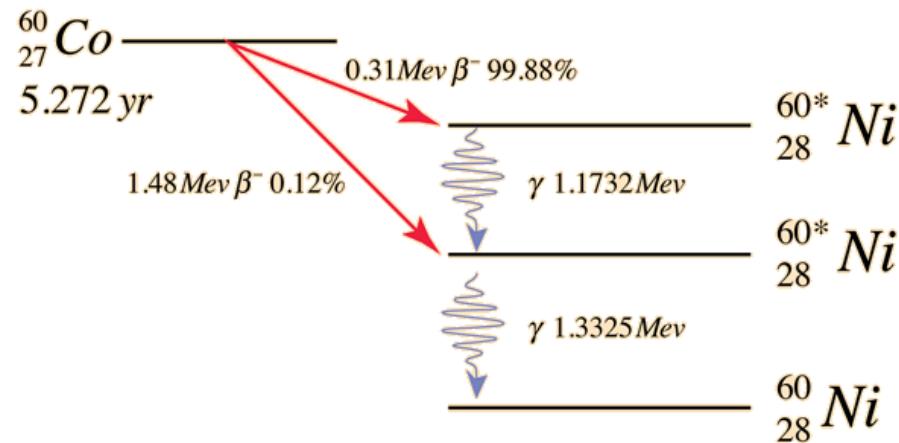
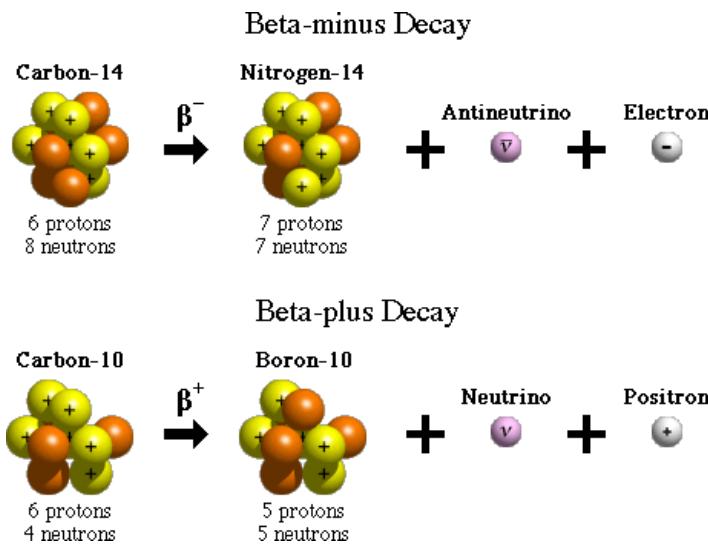


Examples of beta-decay

Nucleus level



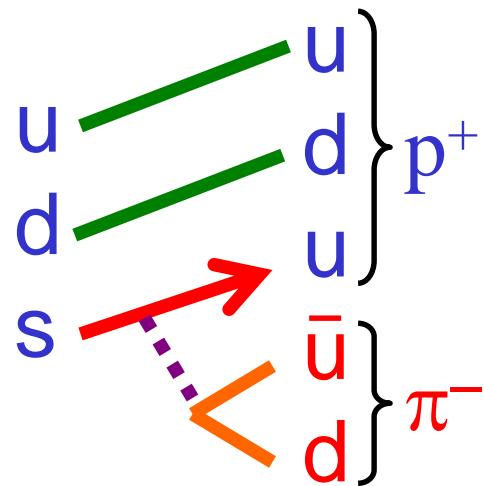
^{40}K , ^{214}Bi , ^3H



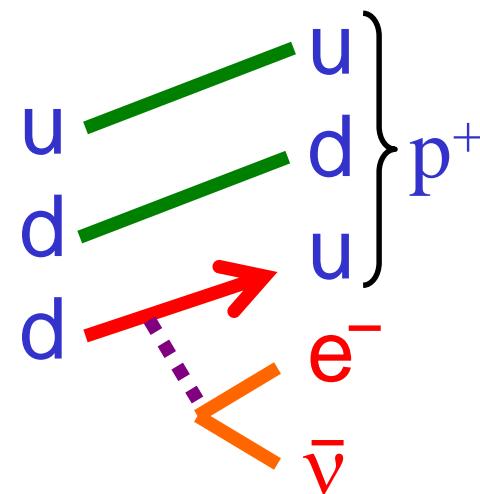
Examples of beta-decay

Barion level (nucleon level)

$$\Lambda^0 \rightarrow p^+ + \pi^-$$

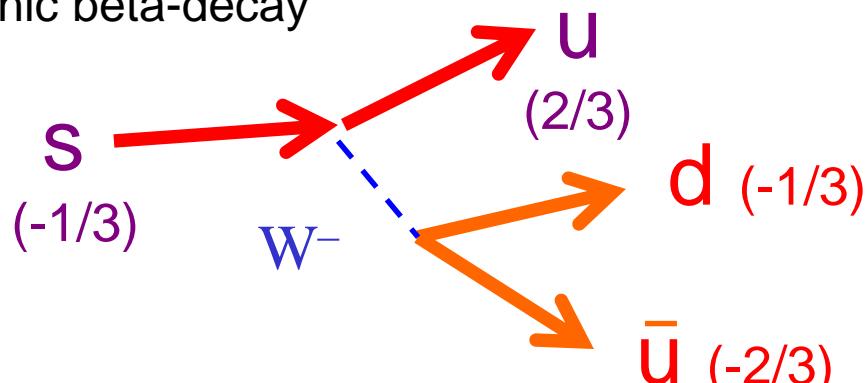


$$n \rightarrow p^+ + e^- + \bar{\nu}$$



Examples of beta-decay

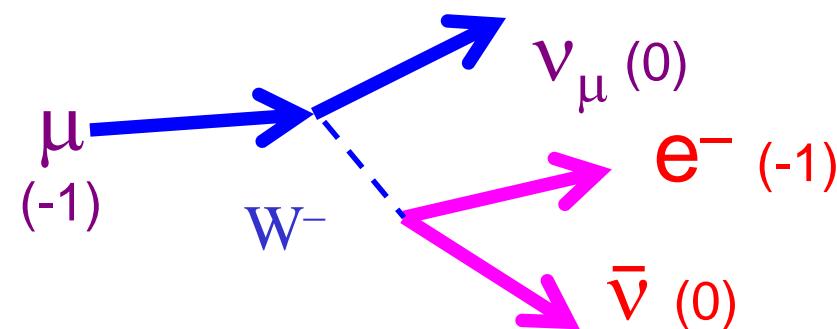
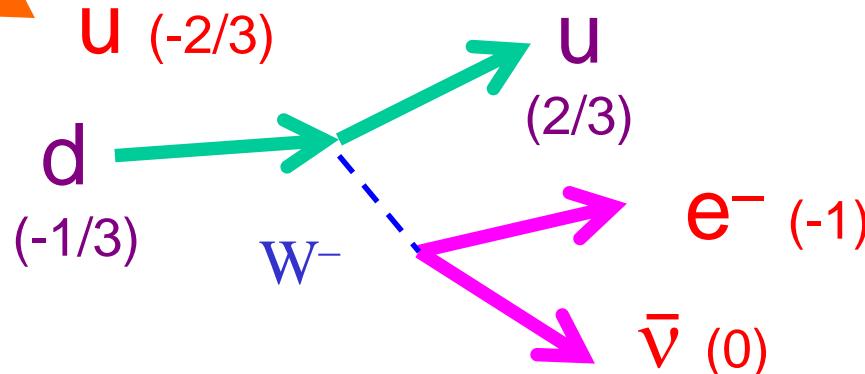
Hadronic beta-decay



Quark level

elementary weak interaction

Semileptonic
beta-decay



Leptonic
beta-decay

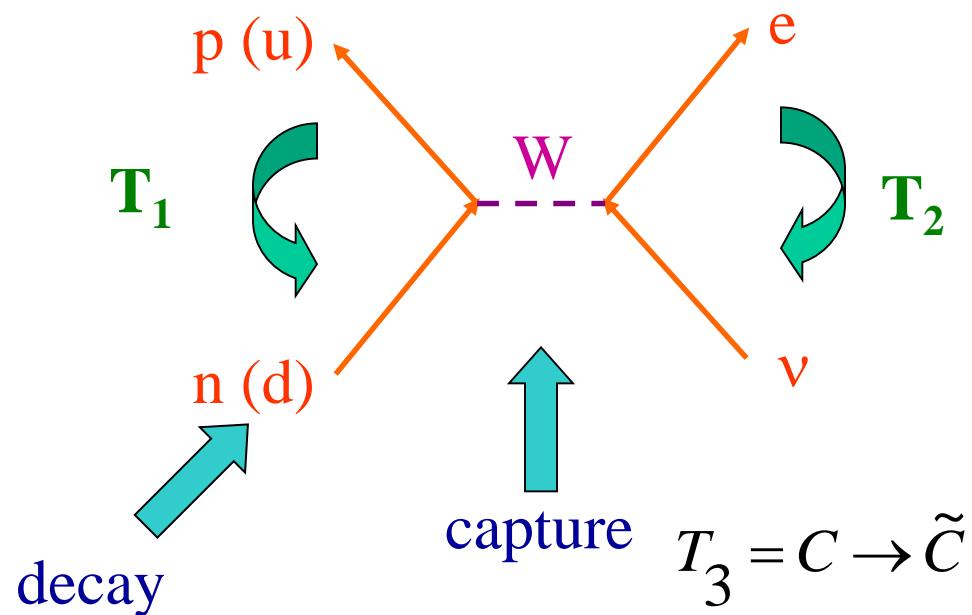
this level = abstract
(no free quarks)

Types of nuclear beta-decay

$$n \rightarrow p^+ + e^- + \tilde{\nu}_e$$
$$p^+ \rightarrow n + e^+ + \nu_e$$

$$e^+ + n \rightarrow p^+ + \tilde{\nu}_e$$
$$e^- + p^+ \rightarrow n + \nu_e$$

$$\nu_e + n \rightarrow p^+ + e^-$$
$$\tilde{\nu}_e + p^+ \rightarrow n + e^+$$



Fermi theory of beta decay

Transition probability

$$w_{k \rightarrow v} = \frac{2\pi}{\hbar} |\langle \Psi_v | H_\beta | \Psi_k \rangle|^2 \varrho(E_v)$$

Interaction matrix element

$$H_{vk} = \langle \Psi_v | H_\beta | \Psi_k \rangle = \int \Psi_v^* H_\beta \Psi_k d\underline{r}^3 d\underline{r}_e^3 d\underline{r}_\nu^3$$

2. Z=0

→ $\Psi_k = \Phi_k(\underline{r})$

→ $\Psi_v = \Phi_v(\underline{r}) \varphi_e(\underline{r}_e) \varphi_\nu(\underline{r}_\nu)$

→ $H_\beta = g \delta(\underline{r} - \underline{r}_e) \delta(\underline{r} - \underline{r}_\nu)$

1. local interaction

$\varphi_e(\underline{r}_e) = N_e e^{-\frac{i}{\hbar} \underline{p}_e \cdot \underline{r}_e}$

$\varphi_\nu(\underline{r}_\nu) = N_\nu e^{-\frac{i}{\hbar} \underline{p}_\nu \cdot \underline{r}_\nu}$

Approximations in the theory

3. long wavelength

$$\underline{p}_e \underline{r} < p_e R$$

$$\frac{p_e R}{\hbar} < \frac{5 \text{MeV} 4 \text{fm}}{\hbar c} \simeq \frac{20}{200} = \frac{1}{10} \ll 1$$

$$e^{\frac{i}{\hbar}(\underline{p}_e + \underline{p}_\nu)\underline{r}} \approx 1 + \frac{i}{\hbar}(\underline{p}_e + \underline{p}_\nu)\underline{r}$$

nuclear matrix element

$$M = M_{kv} = \int \Phi_v(\underline{r})^* \Phi_k(\underline{r}) d\underline{r}^3 = \text{const}(\underline{p}_e)$$

$$M_{kv}^{(1)} = \int \Phi_v(\underline{r})^* \Phi_k(\underline{r}) \frac{i}{\hbar} (\underline{p}_e + \underline{p}_\nu) \underline{r} d\underline{r}^3 = S(\underline{p}_e)$$

4. enabled transition (selection rules)

$$M_{kv} \gg M_{kv}^{(1)}$$

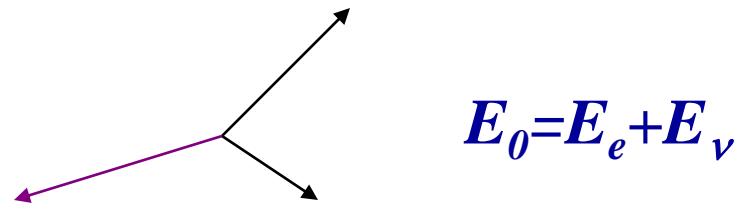
Approximations in the theory

5. recoil of the daughter nucleus is negligible

$$m_M c^2 = m_D c^2 + m_e c^2 + m_\nu c^2 + E_0$$

$$E_0 = E_D + E_e + E_\nu$$

$$E_D = \frac{p_D^2}{2m_D} \ll E_0 \quad m_D \gg m_e$$



6. neutrino mass is negligible

7. direction of the neutrino and the electron are independent

Shape of the energy spectra, FGR

Calculation of the energy spectra

For enabled transition, where $\ell=0$: Φ_v and Φ_k not orthogonal and with $Z=0$ approximation

$$p(E) = w(E) = \text{const} \cdot \rho(E_v)$$

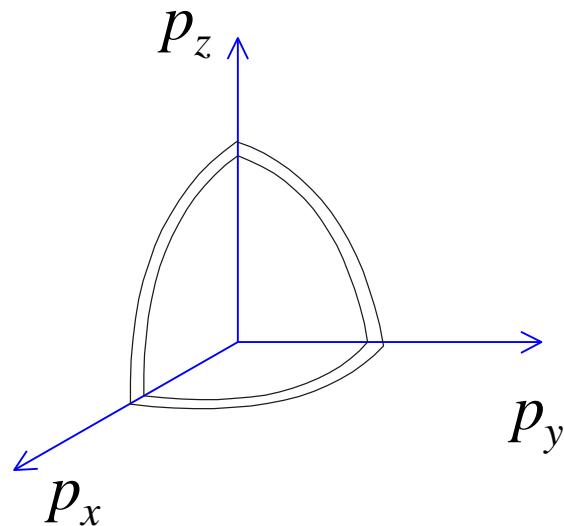
Generally: $p(E) = w(E) = \text{const} \cdot F(Z, E) S(Z, E) \rho(E_v)$

$F(Z, E)$ Fermi function $Z > 0$

$S(Z, E)$ Shape function forbidden transitions

$$\rho(E_v) = \frac{dn}{dE_v} \quad \text{density of states at } E_v \text{ energy of electron}$$

Shape of the spectra, density of states

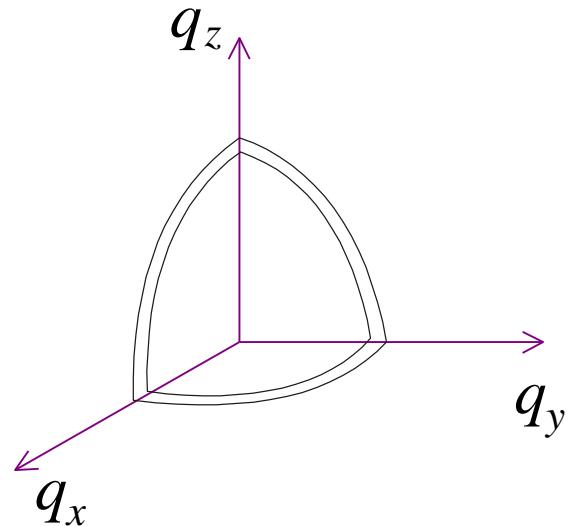


area of the surface $4\pi p^2$

volume of the shell $V=4\pi p^2 dp$

number of states $dN=V/V_1$

density of states $dN/dE=(...) p^2 dp/dE$



there are neutrino states
to each electron state

$$q=E_\nu/c=(E_0-E)/c$$

Independent direction

$$\rho(E) = \frac{dn}{dE} = (...) p^2 \frac{dp}{dE} q^2 \frac{dq}{dE_\nu}$$

Shape of the spectra

$$p^2 = \frac{1}{c^2} \left((E + mc^2)^2 - m^2 c^4 \right)$$

$$q^2 = \frac{E_\nu^2}{c^2}$$

$$\frac{dp}{dE} = \frac{1}{c} \frac{1}{2} \frac{1}{\sqrt{(E + mc^2)^2 - m^2 c^4}} 2(E + mc^2)$$

$$\frac{dq}{dE_\nu} = \frac{1}{c}$$

$$\rho(E) = \frac{dn}{dE} = (\dots) \sqrt{(E + mc^2)^2 - m^2 c^4} (E + mc^2) (E_0 - E)^2$$

$$p(E) = \text{const} \cdot F(Z, E) S(Z, E) p W (W_0 - W)^2$$

$p(E)$ probability of creating an electron with E kinetic energy
 p the momentum of the electron

$W = E + mc^2$ total (relativistic) energy of the electron

$W_0 = E_0 + mc^2$