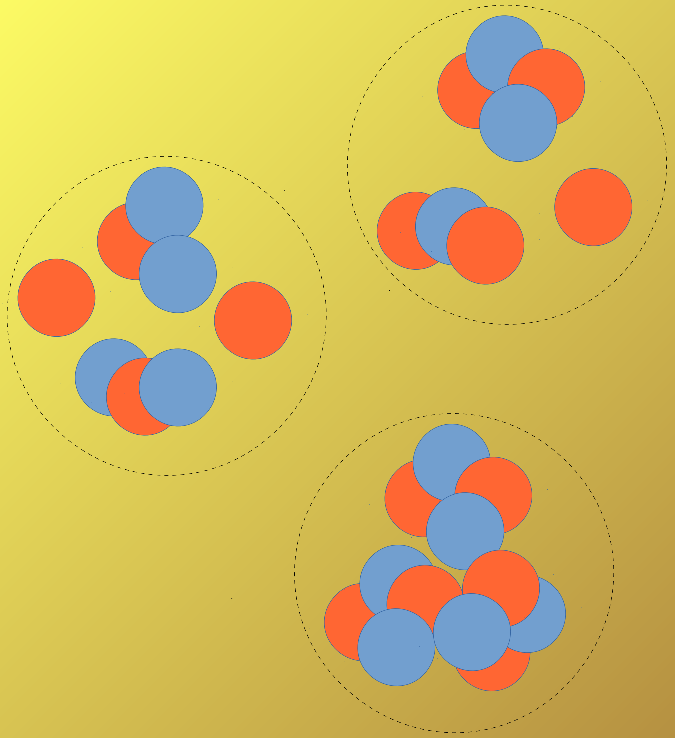
Basics of cluster model with examples

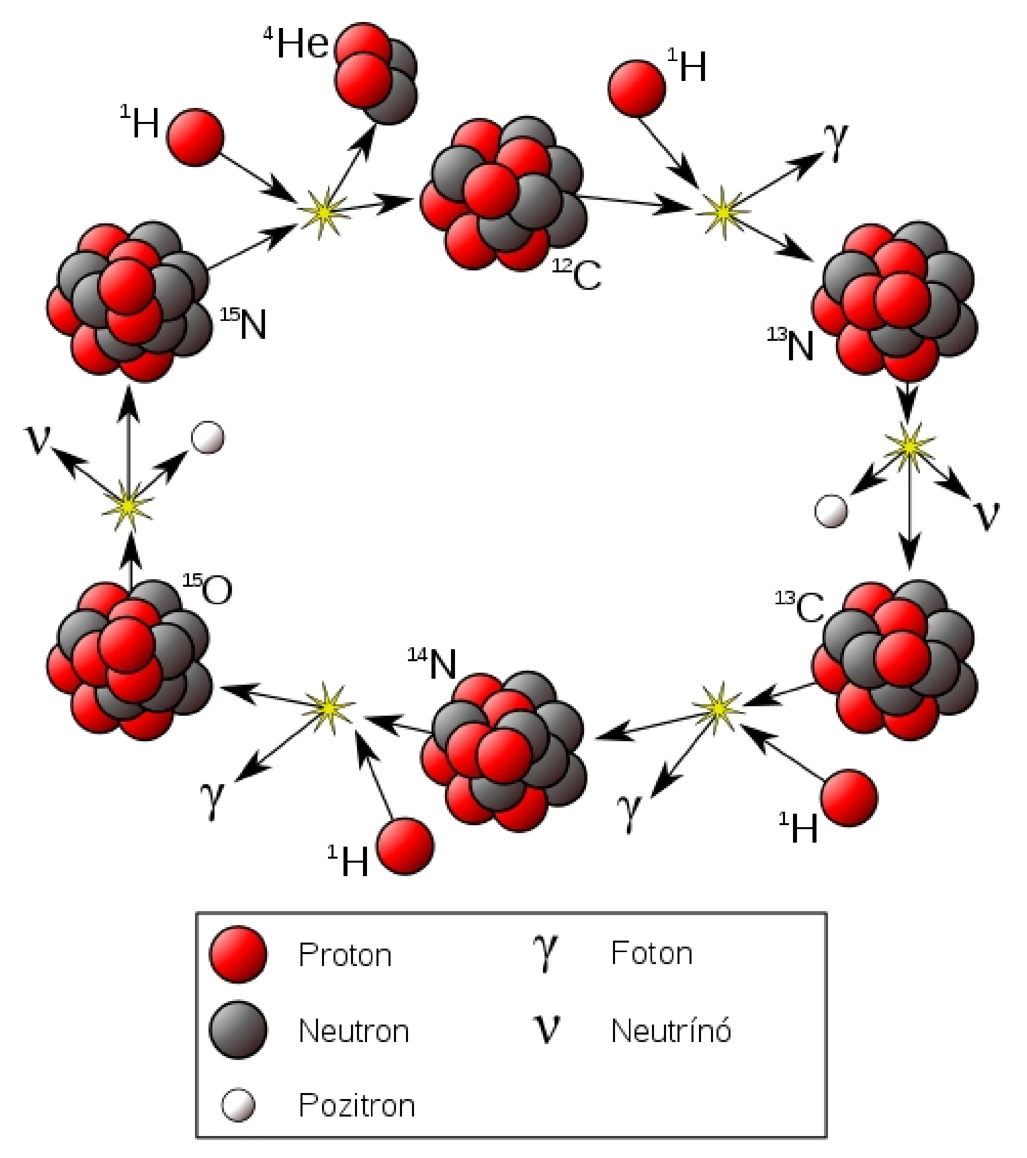


D.Szalkai

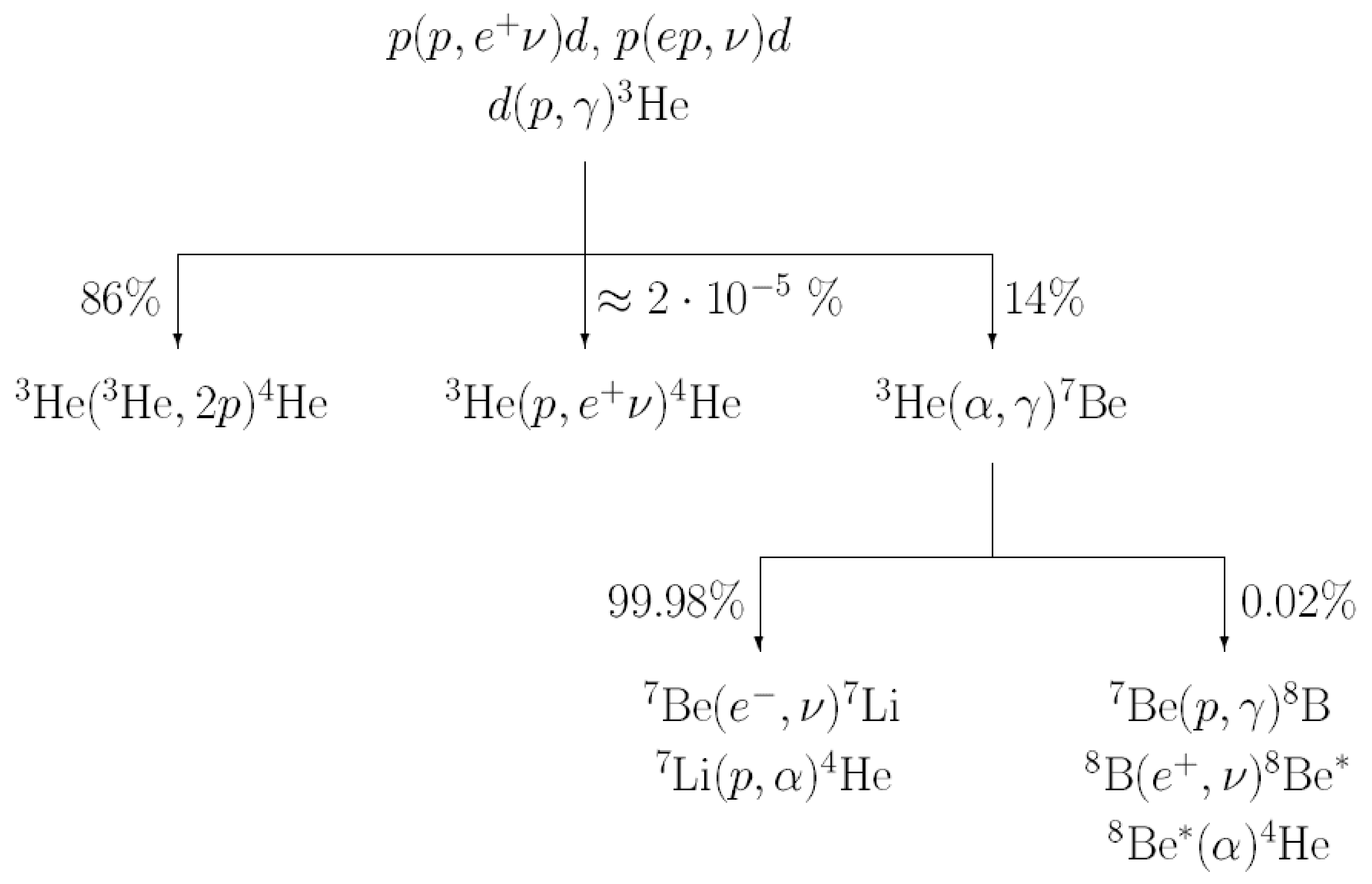
Magfizika szeminárium

2018.11.20

Bigger stars: CNO – cycle



Smaller stars (Sun):
Proton-proton chain reaction



$p(p, e^+\nu)d$, $p(ep, \nu)d$

$d(p, \gamma)^3\mathrm{He}$

86%

$\approx 2 \cdot 10^{-5}$ %

14%

$^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$

$^3\mathrm{He}(p, e^+\nu)^4\mathrm{He}$

$^3\mathrm{He}(\alpha, \gamma)^7\mathrm{Be}$

99.98%

0.02%

$^7\mathrm{Be}(e^-, \nu)^7\mathrm{Li}$

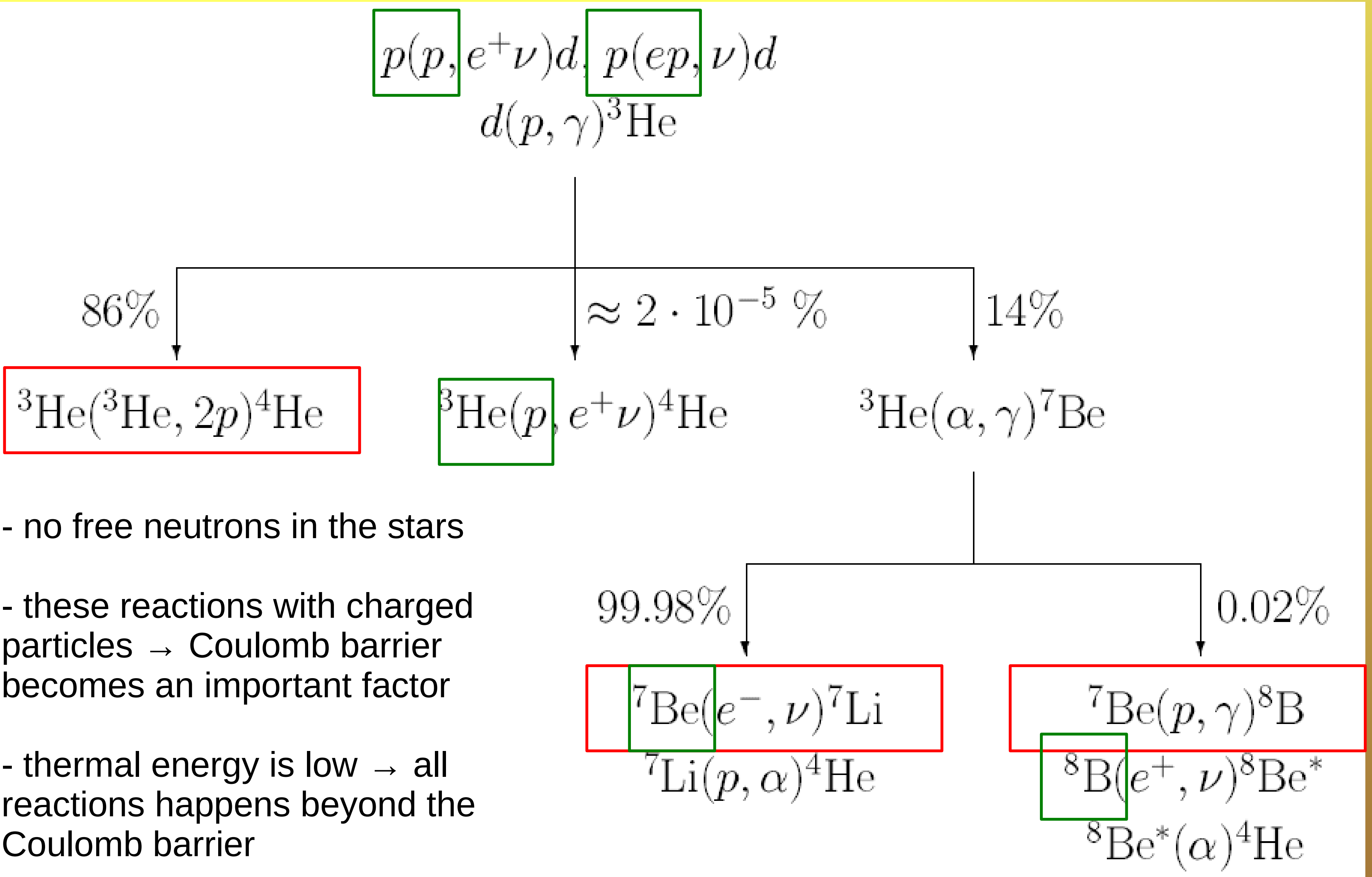
$^7\mathrm{Li}(p, \alpha)^4\mathrm{He}$

$^7\mathrm{Be}(p, \gamma)^8\mathrm{B}$

$^8\mathrm{B}(e^+, \nu)^8\mathrm{Be}^*$

$^8\mathrm{Be}^*(\alpha)^4\mathrm{He}$

Smaller stars (Sun): Proton-proton chain reaction



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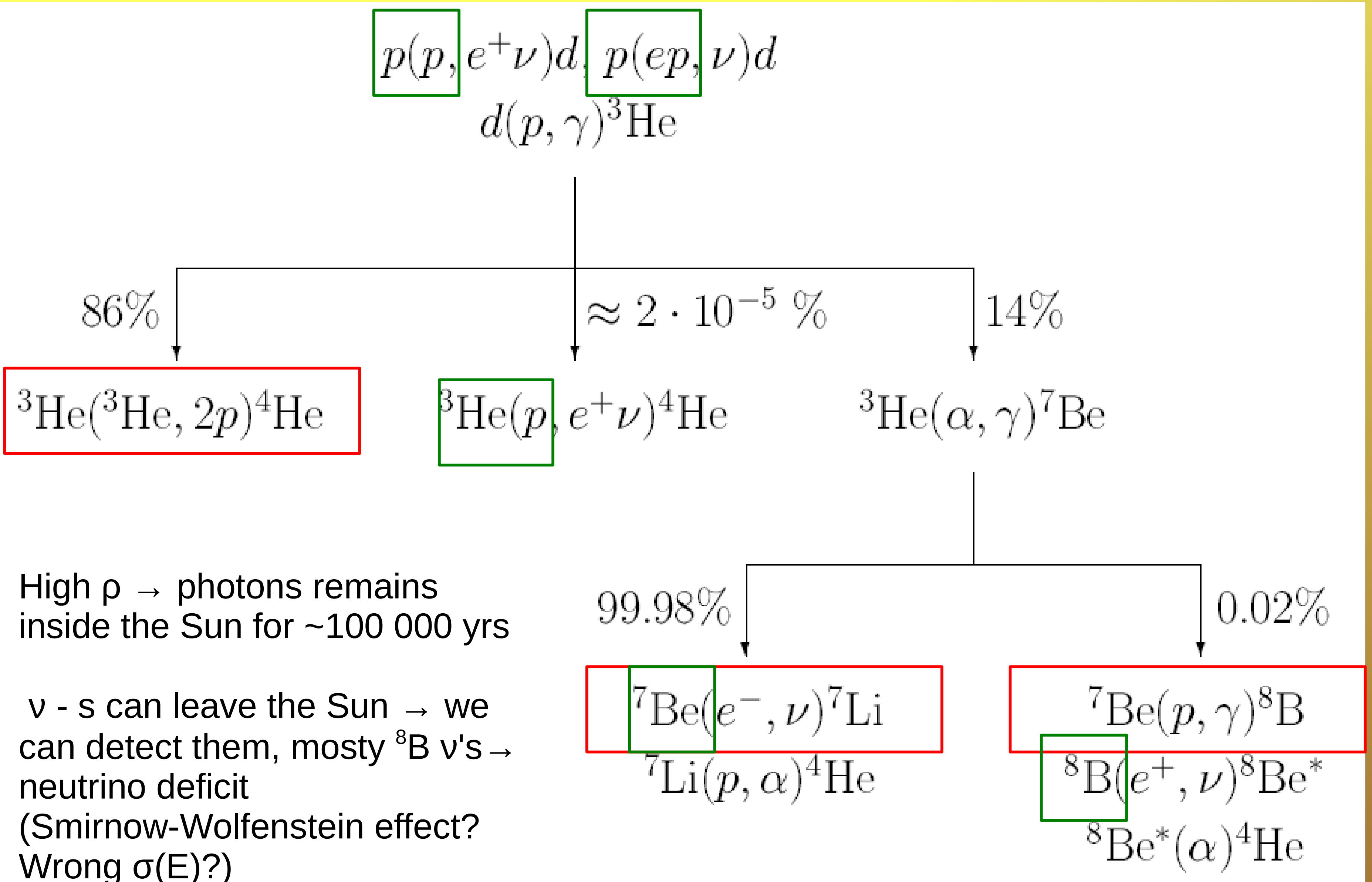
- no free neutrons in the stars

- these reactions with charged particles → Coulomb barrier becomes an important factor

- thermal energy is low → all reactions happens beyond the Coulomb barrier

Smaller stars (Sun): Proton-proton chain reaction



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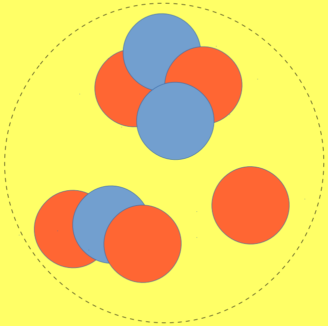
High ρ → photons remains inside the Sun for ~100 000 yrs

ν - s can leave the Sun → we can detect them, mosty ^{8}B ν's → neutrino deficit (Smirnow-Wolfenstein effect? Wrong σ(E)?)

Cluster model

Light atoms: structure + reactions → microscopic dynamical cluster model

N nucleons in A, B, C,... clusters:
Cluster structure of the ^{8}B: A - ^{4}He, B – ^{3}He, C - proton



Wave function in RGM (Resonance group model) for N-body problem

$$\Psi = \mathcal{A}_{AB}\Big(\phi_A^{\rm int}\phi_B^{\rm int}F(\mathbf{r}_{AB})Z(\mathbf{r}_{\rm cm})\Big)$$

Rest anti-symmetrizator between clusters

$$\mathcal{A}_{AB} = \left(\frac{A!B!}{(1+\delta_{AB})N!}\right)^{1/2}\Big(1+\sum_{\varepsilon(A\leftrightarrow B)}(-1)^{\varepsilon}\widehat{P}_{\varepsilon}\Big)$$

Shell model function of inner states of clusters

Function of relative motion of clusters

Function of CM motion

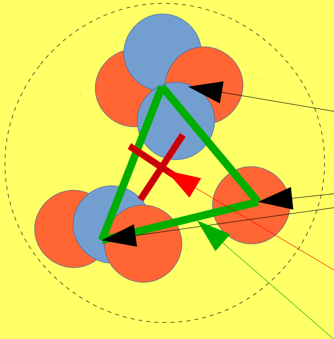
$$\sim \exp\left(-\frac{C\boxed{\beta_C}}{2}\mathbf{r}_C^2\right)$$

Size parameter

$$m\omega_C/\hbar$$

Cluster model

$$\Psi = \mathcal{A}_{AB}\Big(\phi_A^{\rm int}\phi_B^{\rm int}F(\mathbf{r}_{AB})\xcancel{Z(\mathbf{r}}_{\rm cm})\Big)$$



$$\mathbf{r}_C = \frac{1}{C}\sum_{i=1}^{N_C}\mathbf{r}_i$$

CM of clusters

$$\mathbf{r}_{\rm cm} = \frac{A\mathbf{r}_A + B\mathbf{r}_B}{N}$$

Center of nucleus

$$\mathbf{r}_{AB} = \mathbf{r}_A - \mathbf{r}_B.$$

Vector between the CM of clusters

But $\Phi^{\rm int}$ functions depend on internal coordinates
→ Jacobi -coordinates for one-particle coordinates and CM of clusters

$$\{\xi_1^A, \xi_2^A, \ldots, \xi_{N_{A-1}}^A, \xi_1^B, \xi_2^B, \ldots, \xi_{N_{B-1}}^B, \mathbf{r}_{AB}, \mathbf{r}_{\rm cm}\}$$

Fix the quantum states → original N-body problem reduced to the determination of the wave function of the dynamics of relative motion between clusters

^{7}Be(p,ν)^{8}B reaction in cluster model

^{7}Be(p,ν)^{8}B reaction – radiative capture, ^{8}B – positive β-decay
→ unique source of high energy Sun-νs

$$S(E) = \sigma(E) E \exp\left[2\pi\eta(E)\right]$$

Sommerfeld parameter

$$\eta(E) = \frac{\mu Z_1 Z_2 e^2}{k\hbar^2}$$

Scat. wf. δ – scattering phase shift

$$\chi_{\mathrm{s}}(r) \sim \sin(kr + \delta)$$

Bound. wf.
W^{+} Whittaker function, c – asymptotic norm factor

$$\chi_{\mathrm{b}}(r) \sim \bar{c} W^{+}(kr)$$

Experimental data of σ(E)
(proton capture on radioactive ^{7}Be target,
^{8}Be beam – Coulomb dissociation)
EM multipol measurements

Experimental data of S_{17}(E) at 20 keV
19 ± 4,2 – 22.2 ± 2.3 eVb

Model results are different because of the Coulomb barrier…..
Incoming proton encounters the CB already at 250 fm!

σ(E) is sensitive to the wave function of scattering and bound states from 8-10 fm till large distances from the core.

^{7}Be(p,ν)^{8}B reaction in cluster model

$$\boxed{S(E) = \sigma(E) E \exp\left[2\pi\eta(E)\right]}$$

$$\sigma(E) = \sum_{J_i} \frac{1}{(2I_7+1)(2s+1)} \frac{16\pi}{3\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^3 \sum_{l_\omega, I_\omega} (2l_\omega+1)^{-1} |\langle \Psi^{J_f} || \mathcal{M}_1^E || \Psi^{J_i}_{l_\omega, I_\omega}\rangle|^2$$

$$\Psi = \sum_{I_7, I, l_2} \sum_{i=1}^{N_7} \mathcal{A}\left\{\left[\left[\Phi^p_s \Phi^{^7\mathrm{Be},i}_{I_7}\right]_I \chi^i_{l_2}(\boldsymbol{\rho}_2)\right]_{JM}\right\}$$

$$\chi_\mathrm{s}(r) \sim \sin(kr+\delta)$$

$$\chi_\mathrm{b}(r) \sim \bar{c}W^+(kr)$$

$$\Phi^{^7\mathrm{Be},i}_{I_7} = \sum_{j=1}^{N_7} c_{ij} \sum_{l_1} \mathcal{A}\left\{\left[\left[\Phi^\alpha\Phi^h\right]_{\frac{1}{2}} \Gamma^j_{l_1}(\boldsymbol{\rho}_1)\right]_{I_7M_7}\right\}$$

Shell model function of inner states of clusters

$$\Phi^{\mathrm{int}} \sim \exp\left(-\frac{C\beta_C}{2}\mathbf{r}_C^2\right)$$

Size parameter

$$m\omega_C/\hbar$$

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exp(+η)

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Scat. wf. δ – scattering phase shift
at low E δ ~ 0

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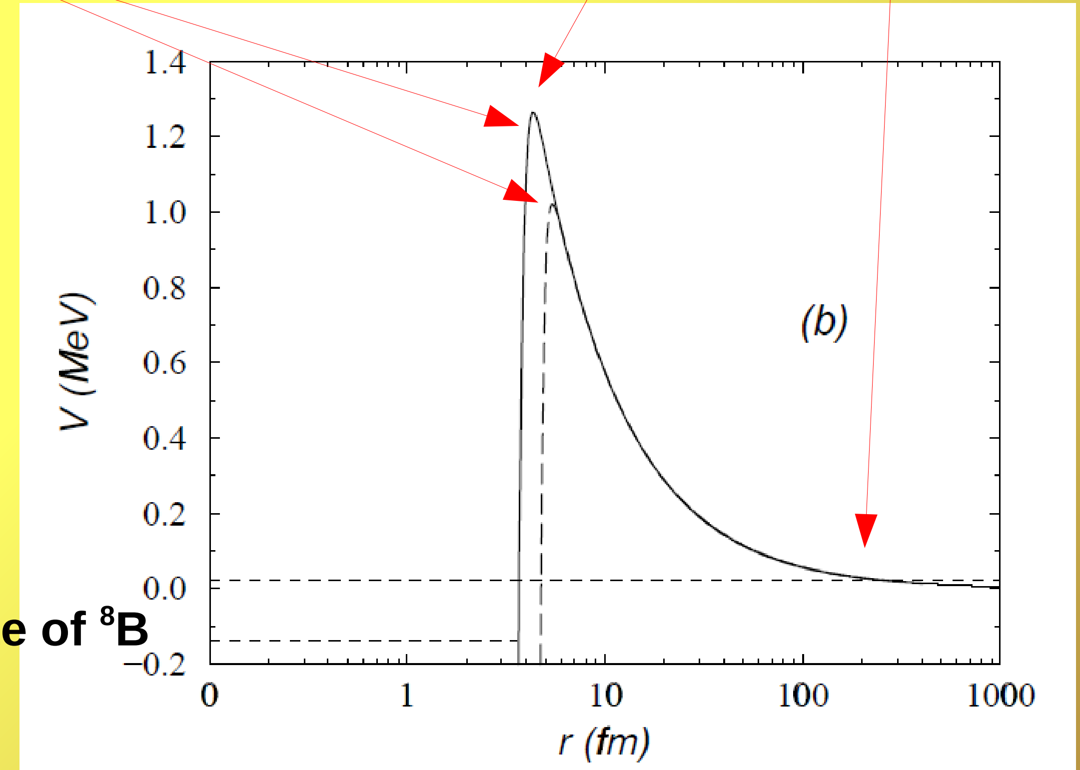
Bound. wf.
W^+ Whittaker function, c – asymptotic norm factor

$$\chi_{\rm b}(r) \sim \bar{c}W^+(kr)$$

Model results are different because of the Coulomb barrier
Incoming proton encounters the CB already at 250 fm!

σ(E) is sensitive to the wave function of scattering and bound states from 8-10 fm till large distances from the core.



20 keV proton

-137 keV bound state of ^{8}B

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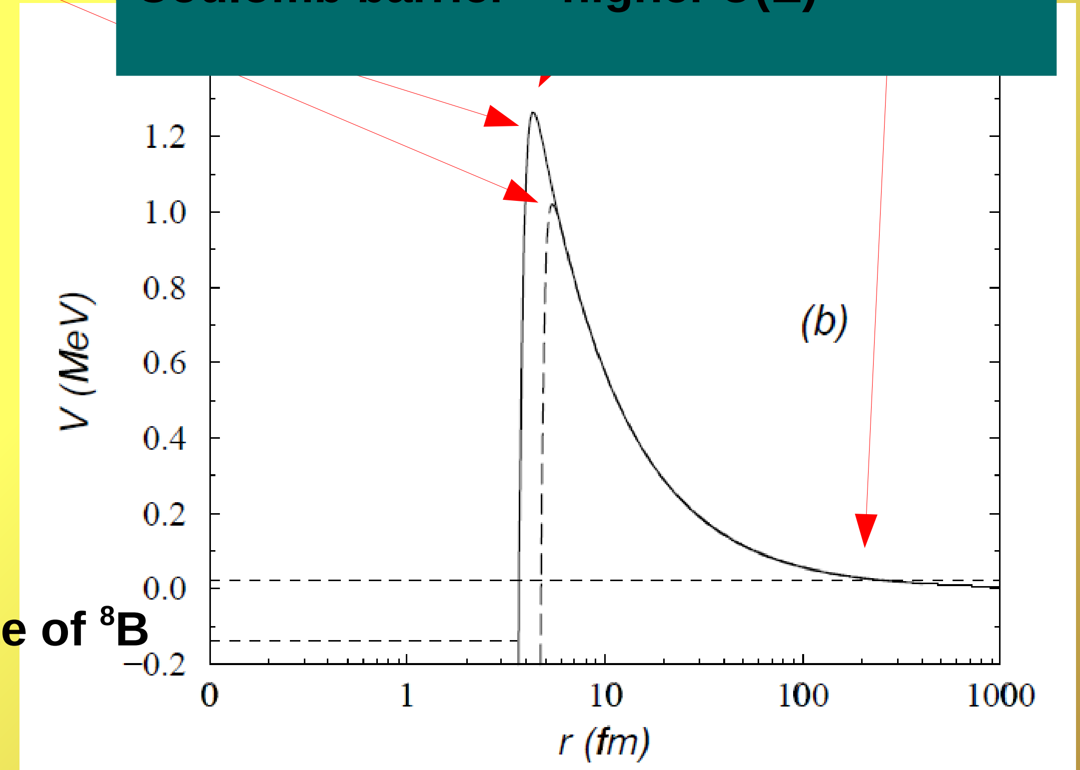
Larger radii of potential – lower Coulomb barrier – higher σ(E)



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Free parameters (^{7}Be(p,ν)^{8}B) : nucleon-nucleon interaction exchange parameter (in Hamilonian) + Size parameter

$$\sigma(E) = \sum_{J_i} \frac{1}{(2I_7+1)(2s+1)} \frac{16\pi}{3\hbar} \left(\frac{E_\gamma}{\hbar c}\right)^3 \sum_{l_\omega, I_\omega} (2l_\omega+1)^{-1} |\langle \Psi^{J_f} || \mathcal{M}_1^E || \Psi^{J_i}_{l_\omega, I_\omega} \rangle|^2$$

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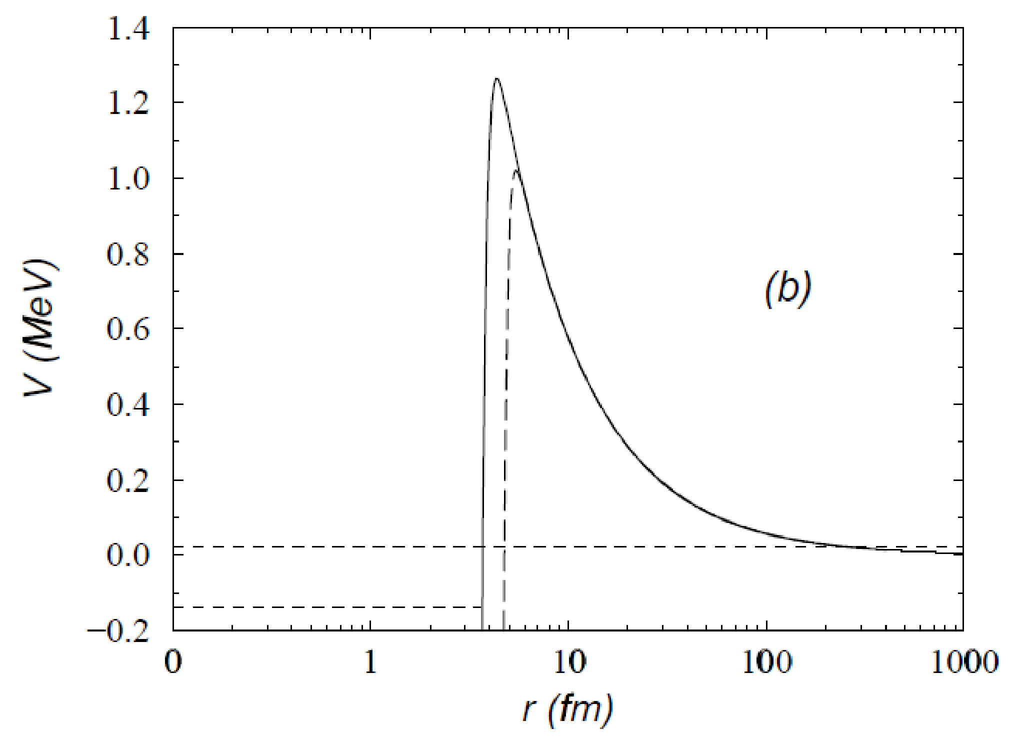

$$\Psi = \sum_{I_7, I, l_2} \sum_{i=1}^{N_7} \mathcal{A}\left\{ \left[\left[\Phi_s^p \Phi_{I_7}^{^7\mathrm{Be},i} \right]_I \chi^i_{l_2}(\boldsymbol{\rho}_2) \right]_{JM} \right\}$$

$$\chi_\mathrm{b}(r) \sim \bar{c} W^+(kr)$$

$$\Phi_{I_7}^{^7\mathrm{Be},i} = \sum_{j=1}^{N_7} c_{ij} \sum_{l_1} \mathcal{A}\left\{ \left[\left[\Phi^\alpha \Phi^h \right]_{\frac{1}{2}} \Gamma^j_{l_1}(\boldsymbol{\rho}_1) \right]_{I_7 M_7} \right\}$$

$$\boldsymbol{\Phi}^{\mathbf{int}} \sim \exp\left(-\frac{C\beta_C}{2} \mathbf{r}_C^2 \right)$$

-137 keV bound state of ^{8}B → this must be reproduced by the model!
→ nucleon-nucleon interaction exchange parameter must be set properly
(LS interaction in ^{8}B: -1/2, -3/2)
Size parameter (β) can be changed →σ(E) ~ β-dependent physical properties

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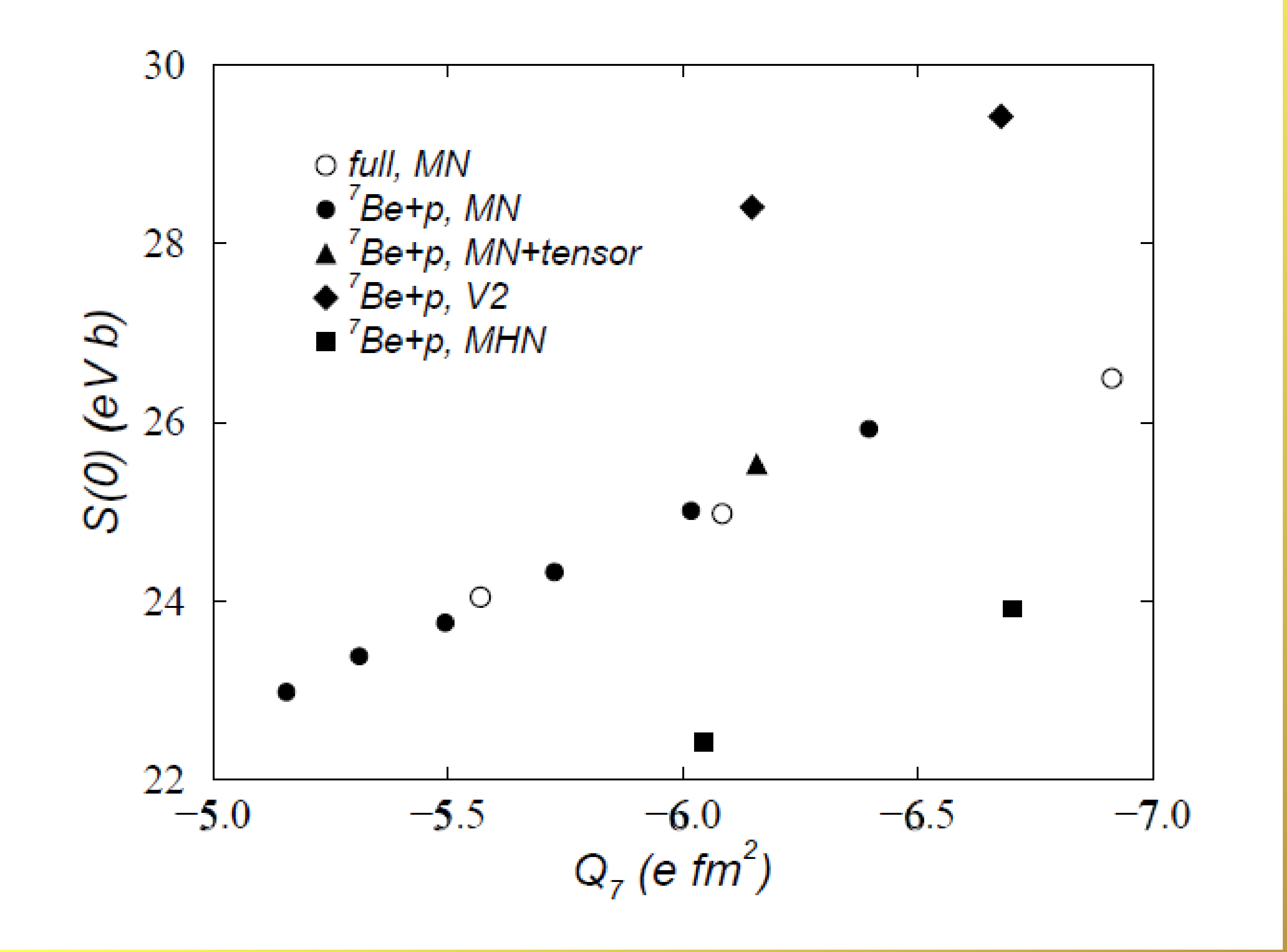
Example: S_{17} – Q_7 (quadrupol moment)

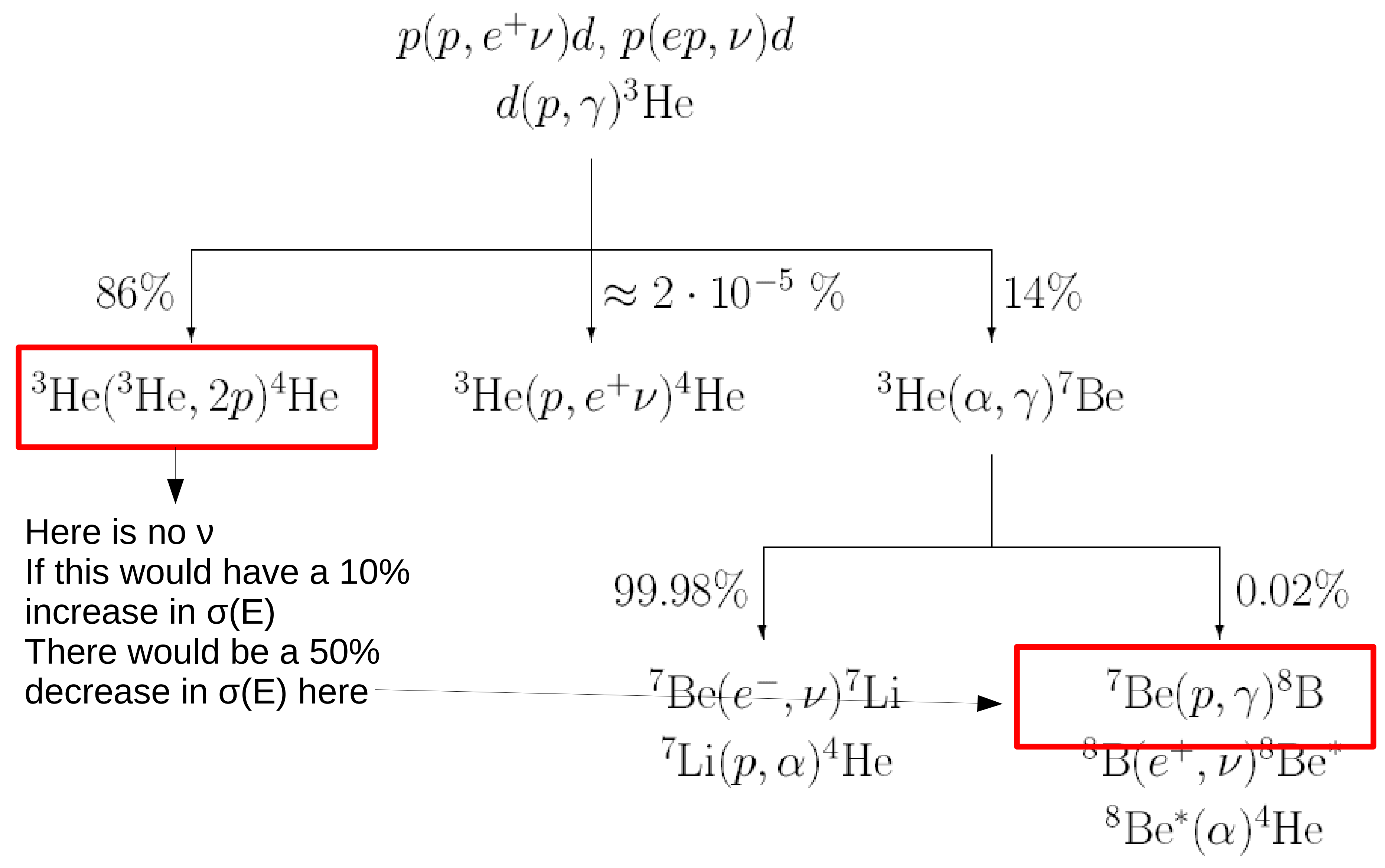


^{8}B Q_8 – experiment: 6.83 ± 0.21 efm2, model: 7.45 efm2

^{7}Be Q_7 – experiment:?, model: -6.9 efm2

S_{17}(0) – experiment: 19 ± 4,2 eVb, model: 26.5 eVb

Smaller stars (Sun): Proton-proton chain reaction



$$p(p,e^+\nu)d,\ p(ep,\nu)d$$

$$d(p,\gamma)^3\mathrm{He}$$

86%

$\approx 2\cdot 10^{-5}\ \%$

14%

$^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$

$^3\mathrm{He}(p,e^+\nu)^4\mathrm{He}$

$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$

Here is no ν

If this would have a 10% increase in σ(E)

There would be a 50% decrease in σ(E) here

99.98%

0.02%

$^7\mathrm{Be}(e^-,\nu)^7\mathrm{Li}$

$^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$

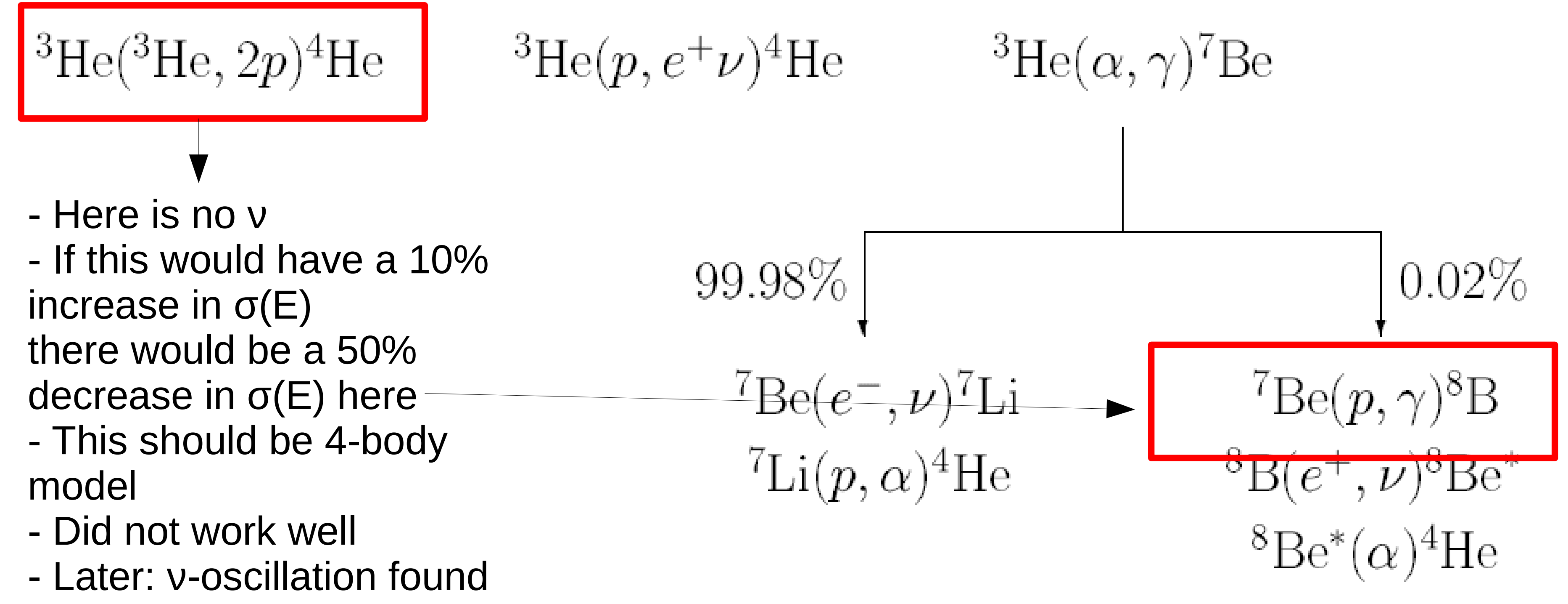
$^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$

$^8\mathrm{B}(e^+,\nu)^8\mathrm{Be}^*$

$^8\mathrm{Be}^*(\alpha)^4\mathrm{He}$

Neutrino flux problem
^{3}He(^{3}He,2p)^{4}He reaction

$$\Psi^{^6\mathrm{Be}} = \sum_{l_1,l_2,L,S} \mathcal{A}\left\{\left[\left[\Phi^\alpha\Phi^p\Phi^p\right]_S \chi^{\alpha pp}_{[l_1,l_2]L}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\right]_{JM}\right\} + \mathcal{A}\left\{\left[\phi^h\phi^h\chi^{hh}_L(\boldsymbol{\rho})\right]\right\}$$



$^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$

$^3\mathrm{He}(p,e^+\nu)^4\mathrm{He}$

$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$

- Here is no ν
- If this would have a 10% increase in σ(E) there would be a 50% decrease in σ(E) here
- This should be 4-body model
- Did not work well
- Later: ν-oscillation found

99.98%

$^7\mathrm{Be}(e^-,\nu)^7\mathrm{Li}$

$^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$

0.02%

$^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$

$^8\mathrm{B}(e^+,\nu)^8\mathrm{Be}^*$

$^8\mathrm{Be}^*(\alpha)^4\mathrm{He}$

Literature

- A. Csótó – Ph.D. “Könnyű atommagok szerkezetének és reakcióinak mikroszkopikus leírása”, 1992
- A. Csótó – D.Sc. "Few-body dynamics in nuclear structure and reactions", 1999