A Nap energiatermelése

Magfizika szeminárium 2019. 10. 03.

Marx Pál Fülöp

Coulomb gát

$$P = exp\left\{-2\left[\frac{2\mu}{\hbar^2}(E_C - E)\right]^{\frac{1}{2}} R_C \left[\frac{arctan\left(\frac{R_C}{R_n} - 1\right)^{\frac{1}{2}}}{\left(\frac{R_C}{R_n} - 1\right)^{\frac{1}{2}}} - \frac{R_n}{R_C}\right]\right\}$$

$$V_C(r) = \frac{Z_1 Z_2 e^2}{r},$$

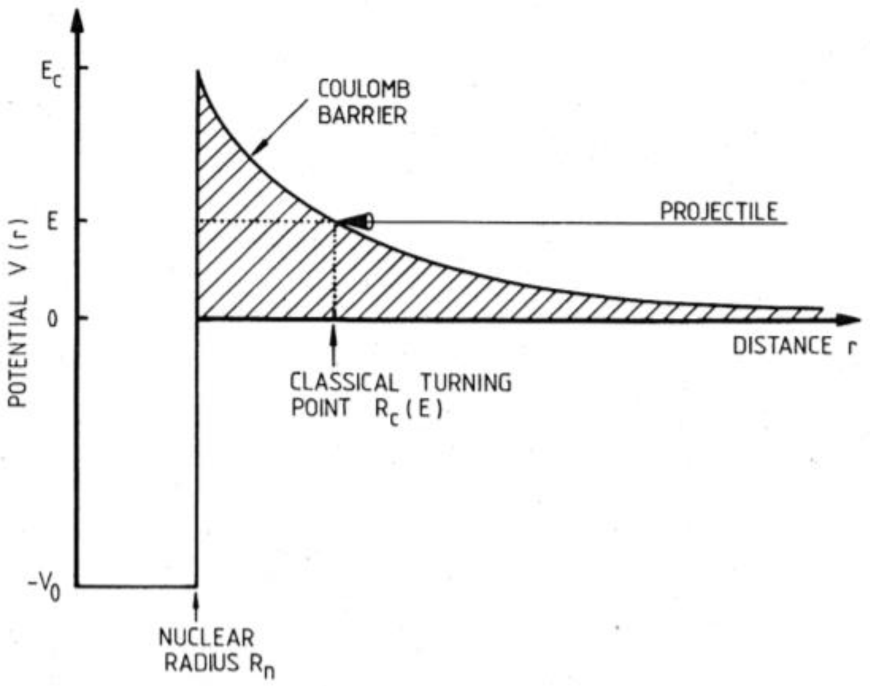
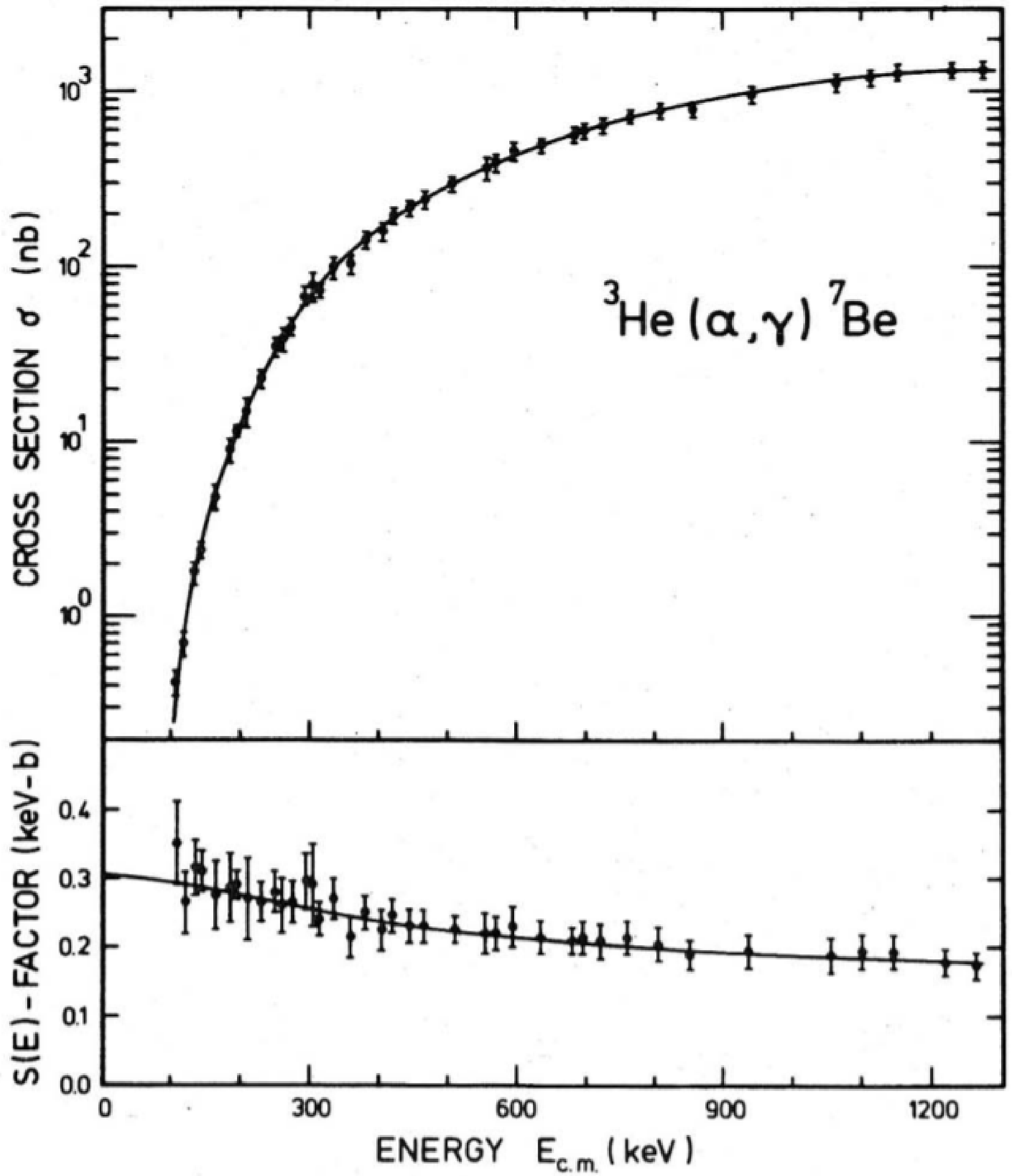


TABLE 4.1 Tunneling Probability for the Reaction $p + p$ as a Function of Center-of-Mass Energy E

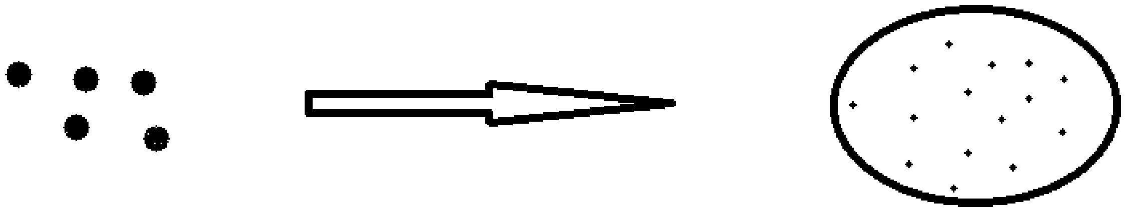
E (keV)	Tunneling Probability: Exact Expression	Tunneling Probability: Approximation
1	8.9×10^{-10}	2.5×10^{-10}
2	5.6×10^{-7}	1.6×10^{-7}
5	1.7×10^{-4}	5.0×10^{-5}
10	3.1×10^{-3}	9.2×10^{-4}
20	2.4×10^{-2}	7.1×10^{-3}
50	0.14	4.4×10^{-2}
100	0.35	0.11
200	0.64	0.21
500	0.99	0'37
550[a]	1.00	0.39

Coulomb gát

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE$$



Reakcióráta



- Lövedék áramsűrűsége: $\frac{N}{A\,dt} = \frac{N\,ds}{A\,ds\,dt} = n_x v$
- Reakcióráta: $n_x n_y \langle \sigma_{(v)} v \rangle$

Hőmérsékleti eloszlás

$$\left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \exp\left(-\frac{\mu v^2}{2kT}\right) v^2 dv \qquad \frac{1}{(kT)^{3/2}} \frac{2}{\sqrt{\pi}} \exp\left(-\frac{E}{kT}\right) \sqrt{E} dE$$

Termikus reakcióráta

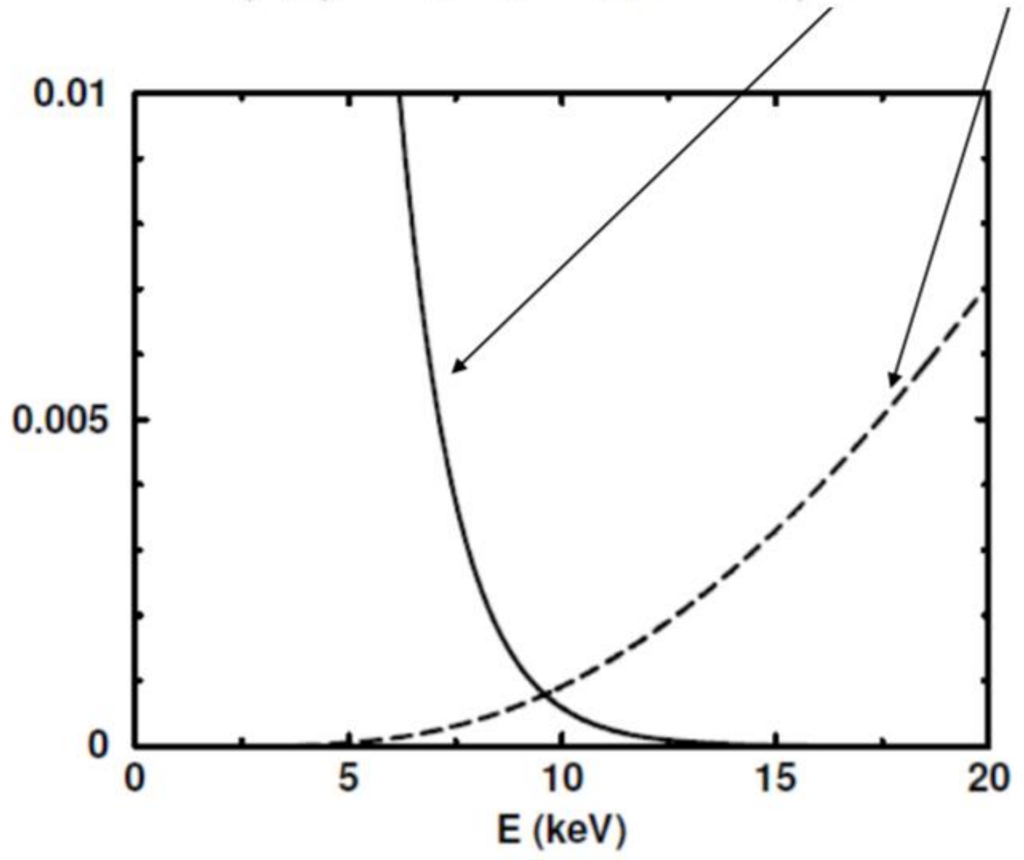
$$n_A n_B \langle \sigma v \rangle = n_A n_B \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \exp\left(-\frac{E}{kT}\right) \sigma(E) dE$$

$$= n_A n_B \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \exp\left(-\frac{E}{kT} - 2\pi\eta(E)\right) S(E) dE$$

Termikus reakcióráta

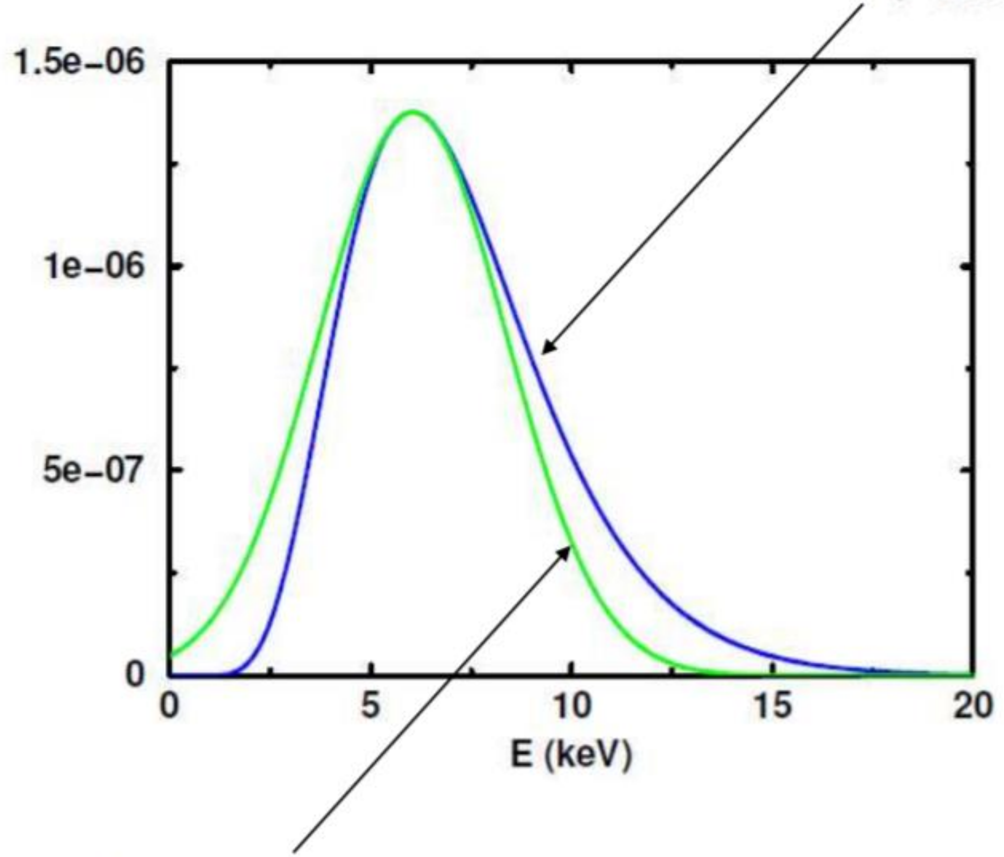
$$n_A n_B \langle \sigma v \rangle = n_A n_B \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \exp\left(-\frac{E}{kT}\right) \sigma(E) dE$$

$$= n_A n_B \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \exp\left(-\frac{E}{kT} - 2\pi\eta(E)\right) S(E) dE$$



Gamow-csúcs

$$\exp\left(-\frac{E}{kT} - 2\pi\eta(E)\right)$$



$I_{\max} \exp(-(E - E_0)^2/(\Delta/2))$ Gauss közelítés

Coulomb gát

$p + p$: $E_0 = 5.9$ keV ,

$p + {}^{14}N$: $E_0 = 26.5$ keV ,

$\alpha + {}^{12}C$: $E_0 = 56$ keV ,

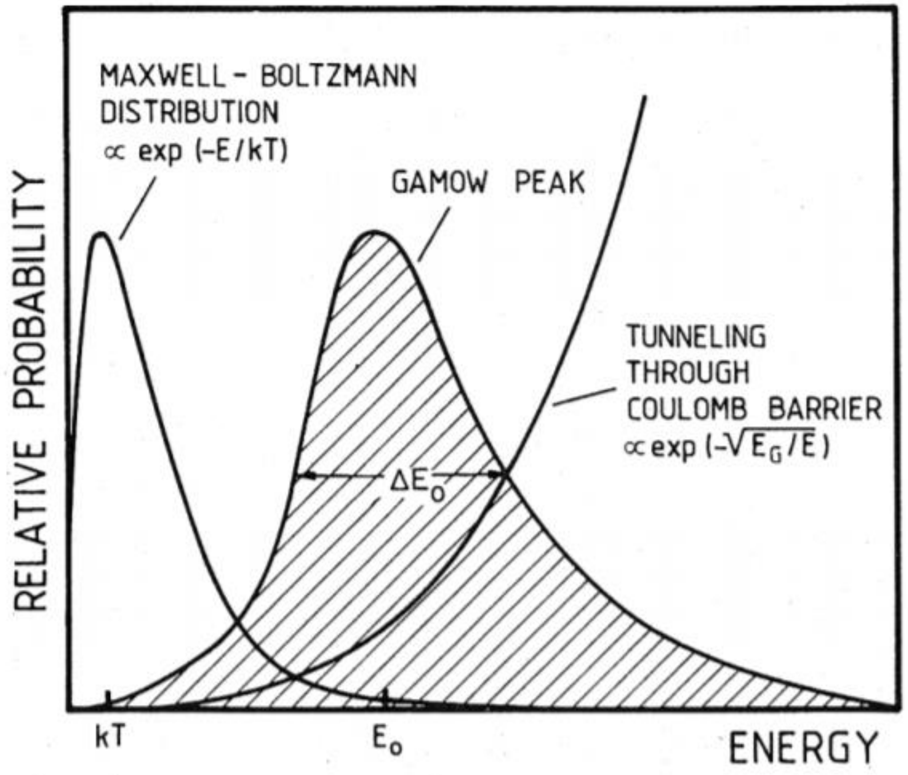
${}^{16}O + {}^{16}O$: $E_0 = 237$ keV .

$p + p$: $I_{max} = 1.1 \times 10^{-6}$,

$p + {}^{14}N$: $I_{max} = 1.8 \times 10^{-27}$,

$\alpha + {}^{12}C$: $I_{max} = 3.0 \times 10^{-57}$,

${}^{16}O + {}^{16}O$: $I_{max} = 6.2 \times 10^{-239}$.

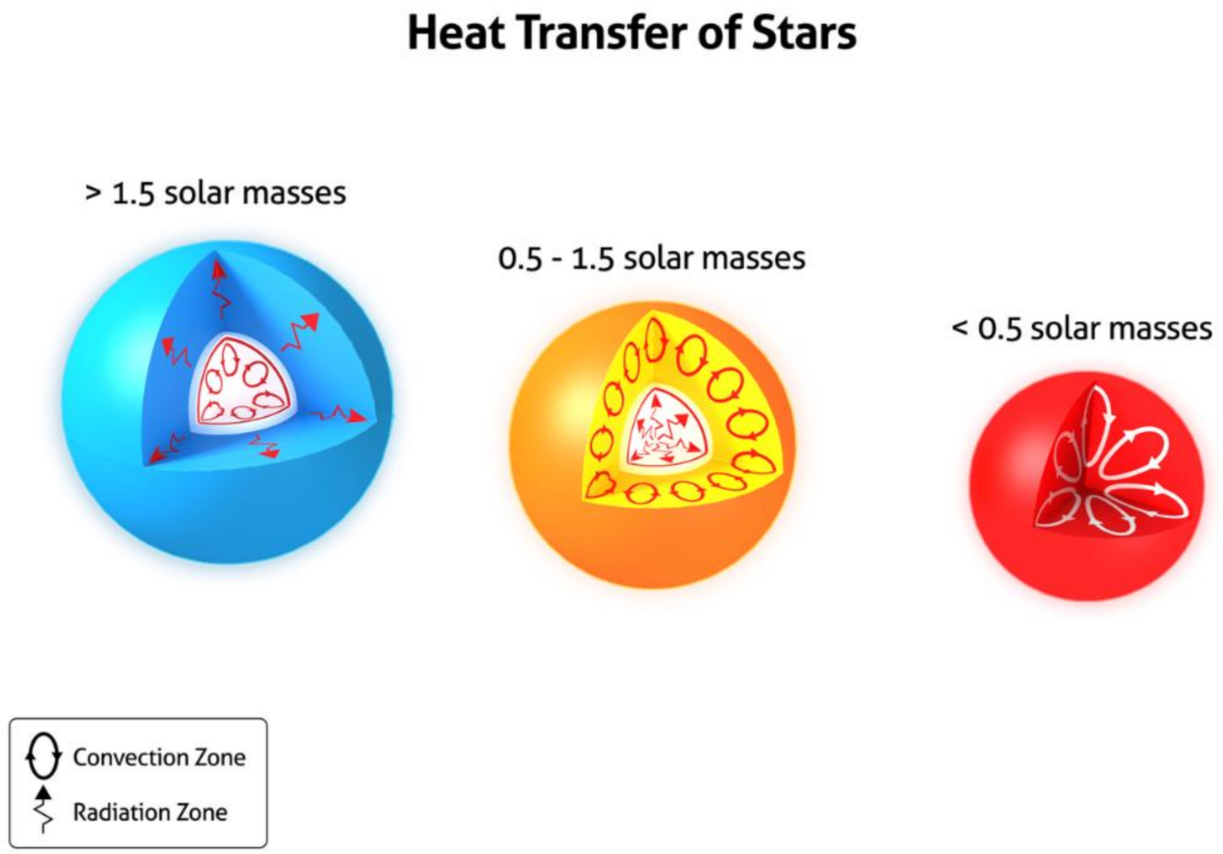


Csillagok szerkezete

A csillag belső szerkezete függ a csillag tömegétől!

$E_* = L * T \approx M$
$L \approx M^{3.5}$

$T \approx M^{-2.5}$



A csillagok egyenletei:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2},$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho.$$

$$\frac{\mathrm{d}l}{\mathrm{d}r} = 4\pi r^2 \rho(\epsilon - \epsilon_\nu),$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{1}{k}\frac{l}{4\pi r^2},$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa\rho l}{64\pi r^2 \sigma T^3},$$

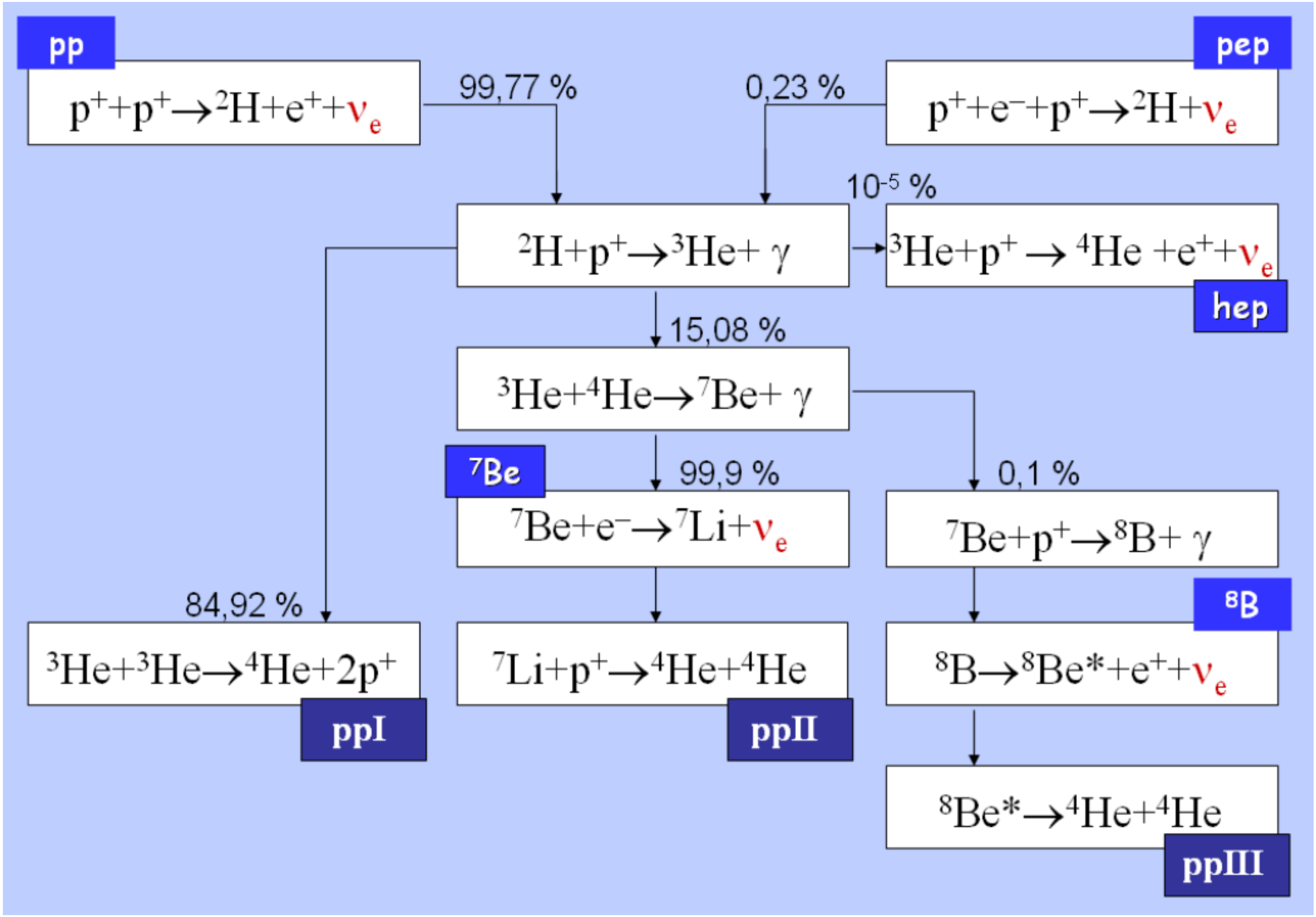
- Sűrűség ρ
- Hőmérséklet T
- Teljes nyomás (anyag és sugárzás) P
- Luminozitás l
- Energiatermelési ráta ε
- Hővezető képesség k
- Opacitás κ
- Stefan-Boltzmann állandó σ
- $k_B = 1$

Proton-proton ciklus

$p + p = {}^2D + e^+ + \nu_e$

$m_p = 938.272\ \mathrm{MeV/c^2}$
$m_D = 1875.612\ \mathrm{MeV/c^2}$
$M_e = 0.511\ \mathrm{MeV/c^2}$
$m_\nu < 0.12\mathrm{eV/c^2}$

$c^2\Delta m = 0{,}421\ \mathrm{MeV}$



CNO-ciklus

Időskála:

$p + p = {}^{2}D + e^{+} + \nu_e$
${}^{14}N + p = {}^{15}O + \gamma$

$$
\begin{aligned}
{}^{12}C + {}^{1}H &\rightarrow {}^{13}N + \gamma \\
{}^{13}N &\rightarrow {}^{13}C + e^{+} + \nu_e \\
{}^{13}C + {}^{1}H &\rightarrow {}^{14}N + \gamma \\
{}^{14}N + {}^{1}H &\rightarrow {}^{15}O + \gamma \\
{}^{15}O &\rightarrow {}^{15}N + e^{+} + \nu_e \\
{}^{15}N + {}^{1}H &\rightarrow {}^{12}C + {}^{4}He
\end{aligned}
$$

Napállandó

A Nap által kisugárzott teljesítmény a Földpálya sugarában:

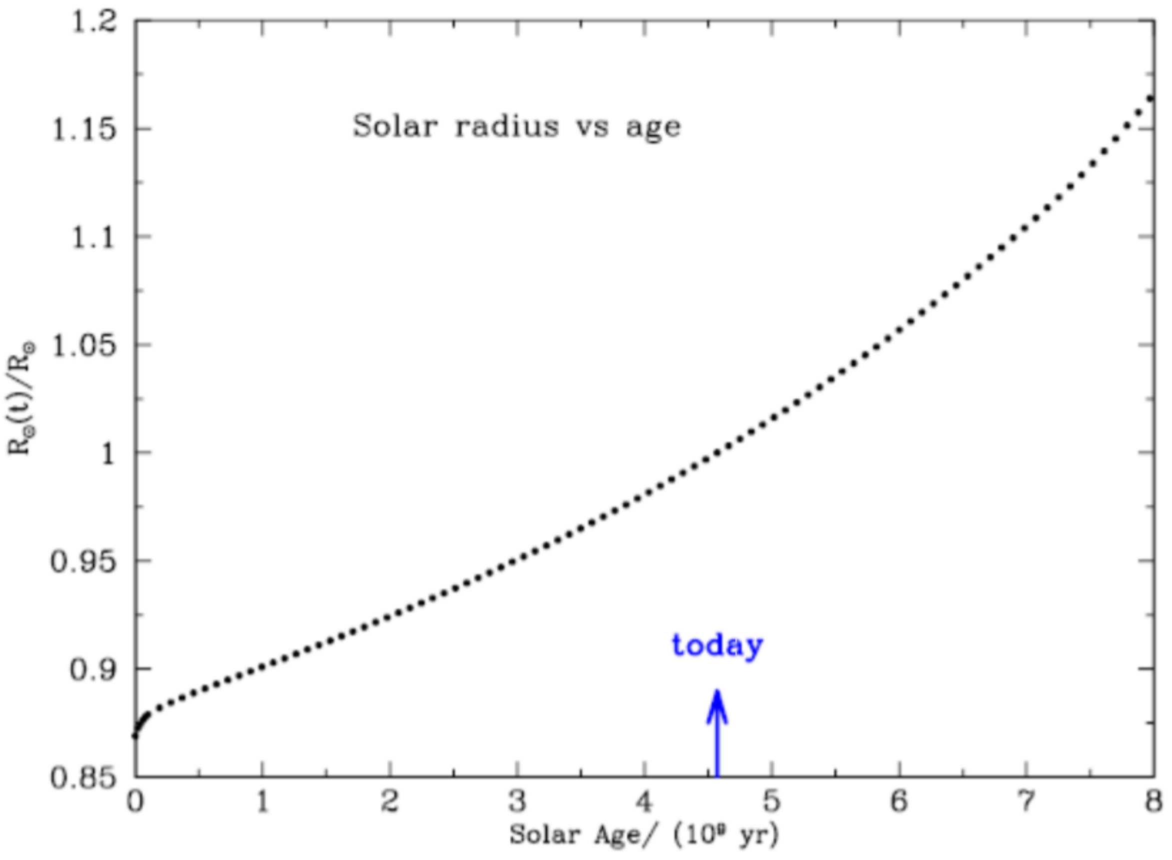
$$k = 1361\ W/m^2$$

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e + 27\ MeV$$

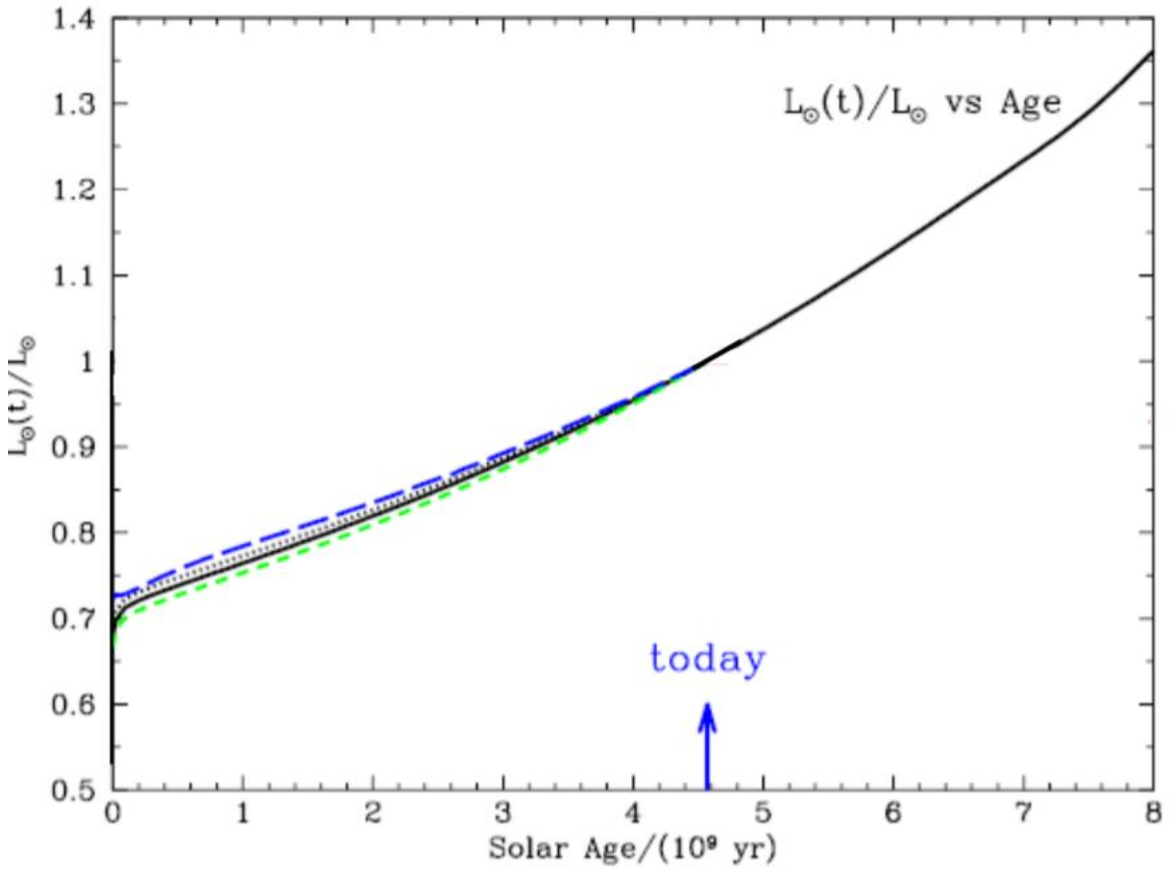
$k4\pi r^2 = \Delta m/\Delta t * c^2$ ⟶ $\Delta m/\Delta t = 7{,}71766\ kg/s$

- Reakció ráta: $R = \frac{k4\pi r^2}{27MeV} = 10^{38}\frac{1}{s}$
- Neutrínó áram: $I = 2 \times 10^{38}\frac{\nu}{s}$
- Neutrínó fluxus: $\frac{I}{4\pi r^2} = 6 \times 10^{10}\frac{\nu}{cm^2 s}$

Feladatok:



Solar radius vs age
today
$R_\odot(t)/R_\odot$
Solar Age/ (10^9 yr)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
6
7
8



$L_\odot(t)/L_\odot$ vs Age
today
$L_\odot(t)/L_\odot$
Solar Age/(10^9 yr)
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
1
2
3
4
5
6
7
8

Hivatkozások

- https://en.wikipedia.org/wiki/Solar_constant
- http://matrix.elte.hu/~csoto/magfizszem/1.pdf
- http://matrix.elte.hu/~csoto/magfizszem/2.pdf
- http://matrix.elte.hu/~csoto/magfizszem/3.pdf
- http://matrix.elte.hu/~csoto/magfizszem/4.pdf
- https://en.wikipedia.org/wiki/Stellar_structure
- http://astro.u-szeged.hu/oktatas/asztrofizika/html/node25.html
- K. Langanke & C. A. Barnes – Nucleosynthesis in the Big Bang and in Stars (World Scientific, Singapore, 1993)

Köszönöm a figyelmet!