HBT-effect and Bose—Einstein correlations

Nuclear physics seminar

September 26, 2019

Roland Pintér

Physics MSc

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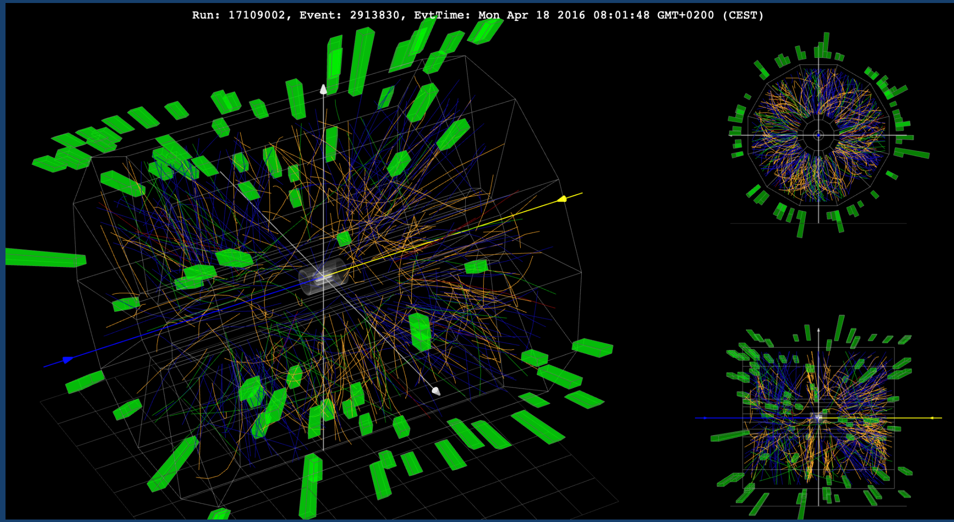
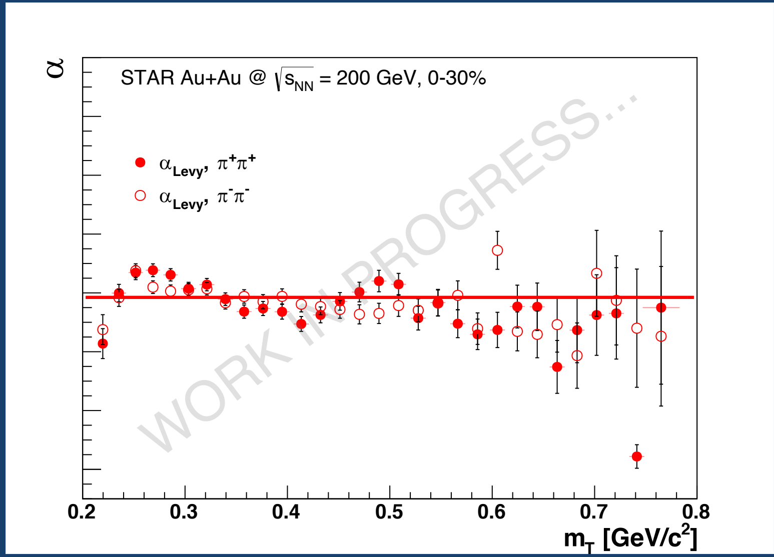
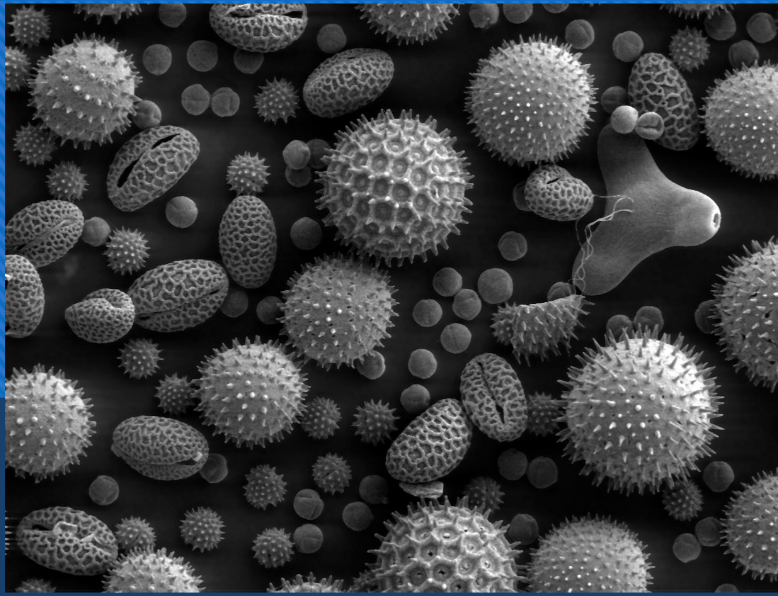


- Radio astronomy – HBT-effect
- High energy physics - Bose–Einstein-correlations
- Results from my research



...scopy



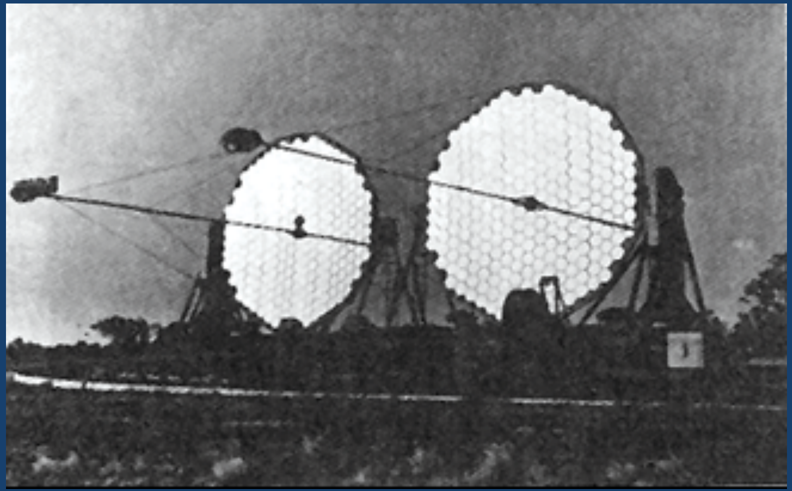
- Wave + object = interference (if $\lambda \sim$ size scale)
- Visible light: $\lambda \in [380, 740]$ nm, 1 nm = 1000 μm → **microscopy** (cannot see smaller objects with microscope)
- Electroscope → **nanoscopy**
  - 1 nm = 10^{-9} m
  - Biological structures
- Nucleus: 1 fm = 10^{-15} m → **femtoscopy**, but with what??

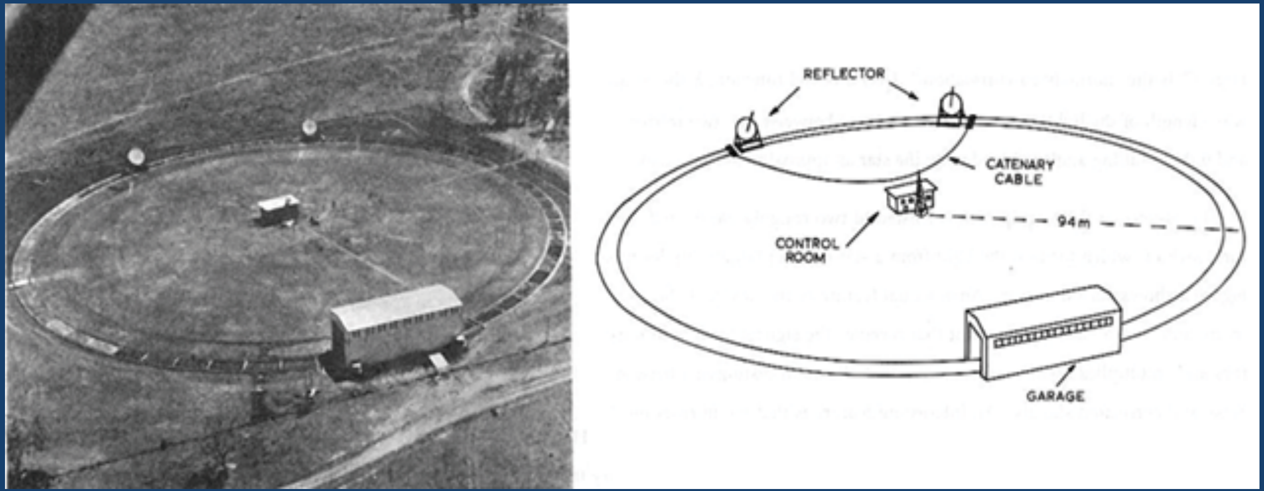
HBT-correlation

- 1931, Karl Guthe Jansky: radio waves emanating from the Milky Way → **radio astronomy**
- R. **H**. **B**rown: observing Sirius with radio telescope
- R. Q. **T**wiss: mathematician, asked by Brown to help with the theory explaining the results

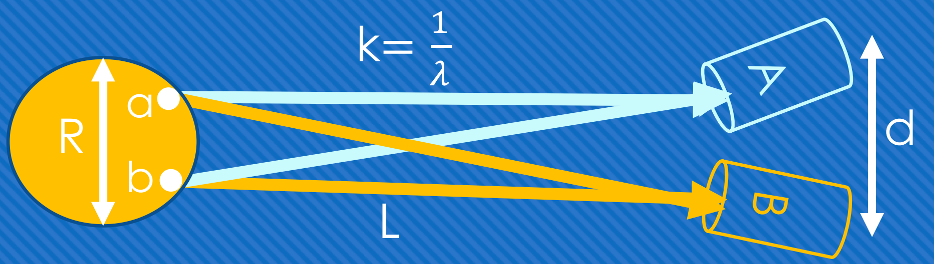
HBT

- Strange **correlation** in the observed data

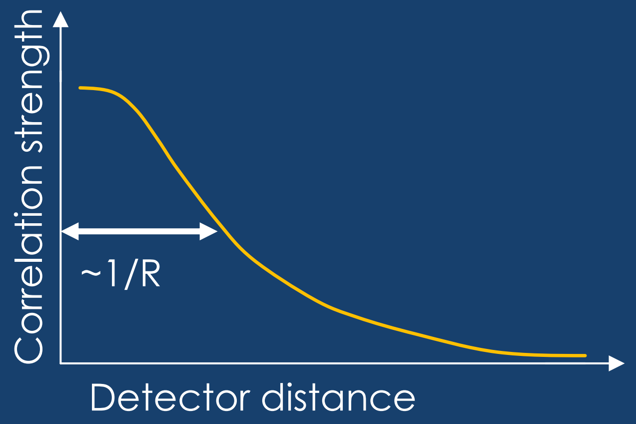




HBT-effect



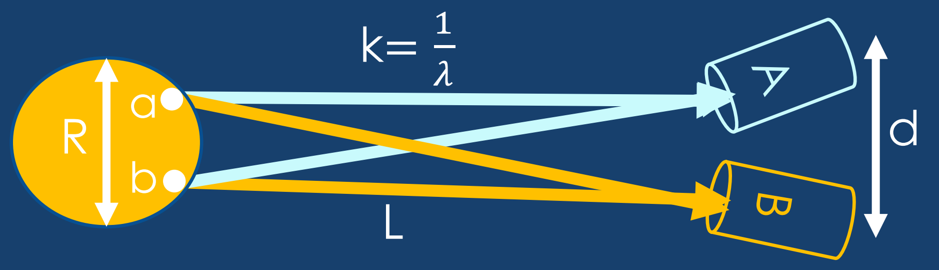
- $\langle I_A \rangle$: average intensity in detector A, from the sources **a** and **b**
- $\langle I_B \rangle$: average intensity in detector B, from the sources **a** and **b**
- $\langle I_A I_B \rangle$: average joint intensity
- Brown's measurement: $C(\delta) - 1 = \frac{\langle I_A I_B \rangle}{\langle I_A \rangle \langle I_B \rangle} - 1 \sim \cos(\delta)$ correlation
  - where $\delta \sim d \cdot R$
- The size of the point-like object (star) is measurable!



HBT with two classical point sources – I.

- The two point sources are chaotic: $\Phi_{a,b}$ random phases
- Spherical waves coming from the two point source:

$$A_{a,b}(r) = \frac{1}{|r-r_{a,b}|}\alpha e^{ik|r-r_{a,b}|+i\phi_{a,b}}$$



- The whole wave arriving at detector A:

$$A(r_A) = A_a(r_A) + A_b(r_A) \cong \frac{1}{L}(\alpha e^{ikr_{aA}+i\Phi_a} + \beta e^{ikr_{bA}+i\phi_b})$$

- Where the intensity:

$$I_A = |A(r_A)|^2 \cong \frac{1}{L^2}(|\alpha|^2 + |\beta|^2 + \alpha^*\beta e^{ik(r_{bA}-r_{aA})+i(\Phi_b-\Phi_a)} + c.c.)$$

- In its time average in the case of chaotic (thermal, random phase) radiation the phases are disappearing, thus:

$$\langle I_A\rangle = \langle I_B\rangle = \frac{1}{L^2}(|\alpha|^2 + |\beta|^2)$$

HBT with two classical point sources – II.

- The time average of the product of the intensities:

$$\langle I_A I_B \rangle = \langle |A(r_A)|^2 |A(r_B)|^2 \rangle$$

- The momentary amplitude square:

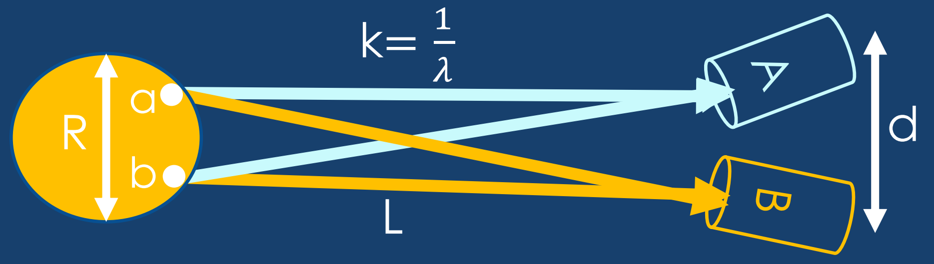
$$|A(r_A)|^2 \cong \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^* \beta e^{ik(r_{bA} - r_{aA}) + i(\Phi_b - \Phi_a)} + c.c.\right)$$

- Thus the phases in 1-1 parts are disappering, and we get:

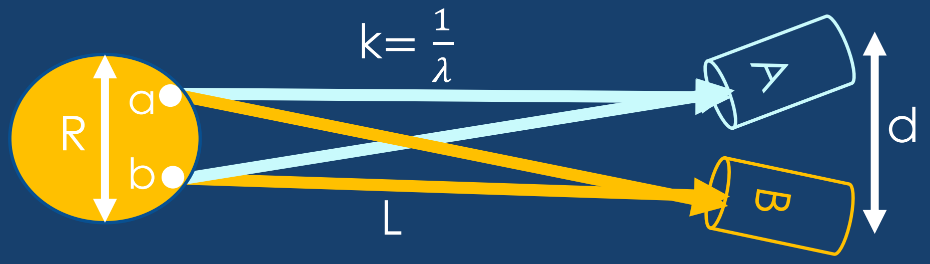
$$\langle I_A I_B \rangle = \frac{1}{L^4}\left(|\alpha|^2 + |\beta|^2\right)^2 + \frac{2}{L^4}|\alpha|^2 |\beta|^2 cos(k[r_{aA} - r_{bA} + r_{aB} - r_{bB}])$$

- Geometry: $k(r_{aA} - r_{bA} + r_{aB} - r_{bB}) \approx \frac{kRd}{L} = R \cdot \Delta k$
- In case of: $\alpha = \beta$ and $d, R \ll L$:

$$\boxed{C_{AB}(\delta) - 1 = \frac{\langle I_A I_B \rangle}{\langle I_A \rangle \langle I_B \rangle} - 1 = \frac{1}{2}\cos(R \cdot \Delta k)}$$



HBT with two quantum sources



- One-particle wave functions:

$$\Psi_{a,b}(r) = \frac{1}{|r-r_{a,b}|} e^{ik|r-r_{a,b}|+i\Phi_{a,b}}$$

- Two-particle wave function:

$$\Psi_{A,B}(r) = \Psi(R_A, R_B) = \frac{1}{\sqrt{2}}(\Psi_a(R_A)\Psi_{\mathrm{b}}(\mathrm{R_B}) + \Psi_a(R_B)\Psi_b(R_A))$$

- The average two-particle probability:

$$\left\langle \left|\Psi_{A,B}\right|^2 \right\rangle = \frac{1}{L^4}(1 + \cos(R \cdot \Delta k))$$

- The result is similar to the classical case:

$$C_{AB} - 1 = \frac{\left\langle \left|\Psi_{A,B}\right|^2 \right\rangle}{\langle|\Psi_a|^2\rangle\langle|\Psi_b|^2\rangle} - 1 = \cos(R \cdot \Delta k)$$

- However, the ½ multiplier is missing!

Femtoscopy with extensive sources

- What happens in case of an extensive source, with $S(r)$ distribution?

$$\Psi(r) = e^{ikr}, \quad \Psi_2(r_1, r_2) = \frac{1}{\sqrt{2}}\left(e^{ik_1r_1}e^{ik_2r_2} + e^{ik_1r_2}e^{ik_2r_1}\right)$$

$$|\Psi(r)|^2 = 1, \quad |\Psi_2(r_1, r_2)|^2 = 1 + \cos[(k_1 - k_2)(r_1 - r_2)]$$

$$N_1(k) = \int S(r,k)|\Psi(r)|^2 d^4r = \int S(r,k) d^4r$$

$$N_2(k_1, k_2) = \int S(r_1, k_1) S(r_2, k_2) |\Psi_2(r_1, r_2)|^2 d^4r_1 d^4r_2$$

$$C_2(k_1, k_2) = \frac{N_2(k_1,k_2)}{N_1(k_1)N_2(k_2)} \cong 1 + \left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2 \quad \text{where } q = k_1 - k_2,\ K = \frac{k_1+k_2}{2}$$

Notations:
- k: momentum
- q: relative momentum
- K: pair's average momentum
- $N_1(k)$: one-particle invariant momentum distribution
- $N_2(k_1, k_2)$: two-particle invariant momentum distribution

- With normed source: $C(q) = 1 + \left|\tilde{S}(q)\right|^2$ where $\tilde{S}(q) = \int S(r) e^{iqr}$
- Can be inverted, we can get $S(r)$ from $C(q)$
- Approximations: no interaction (plain wave), thermal emission (random phases)

Source or correlation function?

Notations:
-ρ: pair's center of mass
-$r = r_1 - r_2$: pair's spatial separation

- With approximations (no interaction, thermal emission):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2 \cong 1 + \left|\int S(r,K) e^{iqr} dr\right|^2$$

- Let's introduce the spatial correlation function:

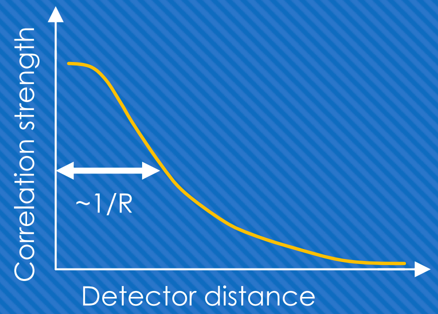
$$D(r,K) = \int S\left(\rho+\frac{r}{2}, K\right) S\left(\rho-\frac{r}{2}, K\right) d\rho$$

- With this, the Bose—Einstein-correlation function:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K) e^{iqr} dr = 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}$$

- The Bose—Einstein-correlation function measures the spatial correlation!

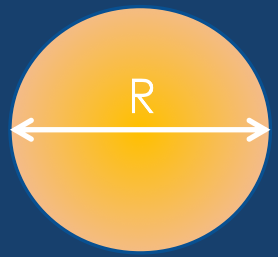
To sum up femtoscopy



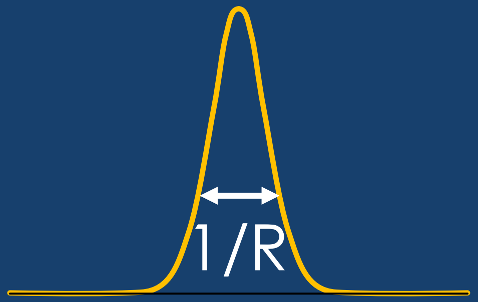
- R. H. Brown, R. Q. Twiss: observed Sirius with radio telescopes
  - Intesity correlation VS detector distance → source size
  - Measured the size of point-like sources
- Goldhaber et al: application in high energy physics
  - Momentum correlation $C(q)$ related to the source $S(r)$
  - $C(q) \cong 1 + \left|\int S(r) e^{iqr} dr\right|^2$ (with some approximations)

  or the distance distribution $D(r)$

  $C(q) \cong 1 + \int D(r) e^{iqr} dr$



$S(r)$ source function

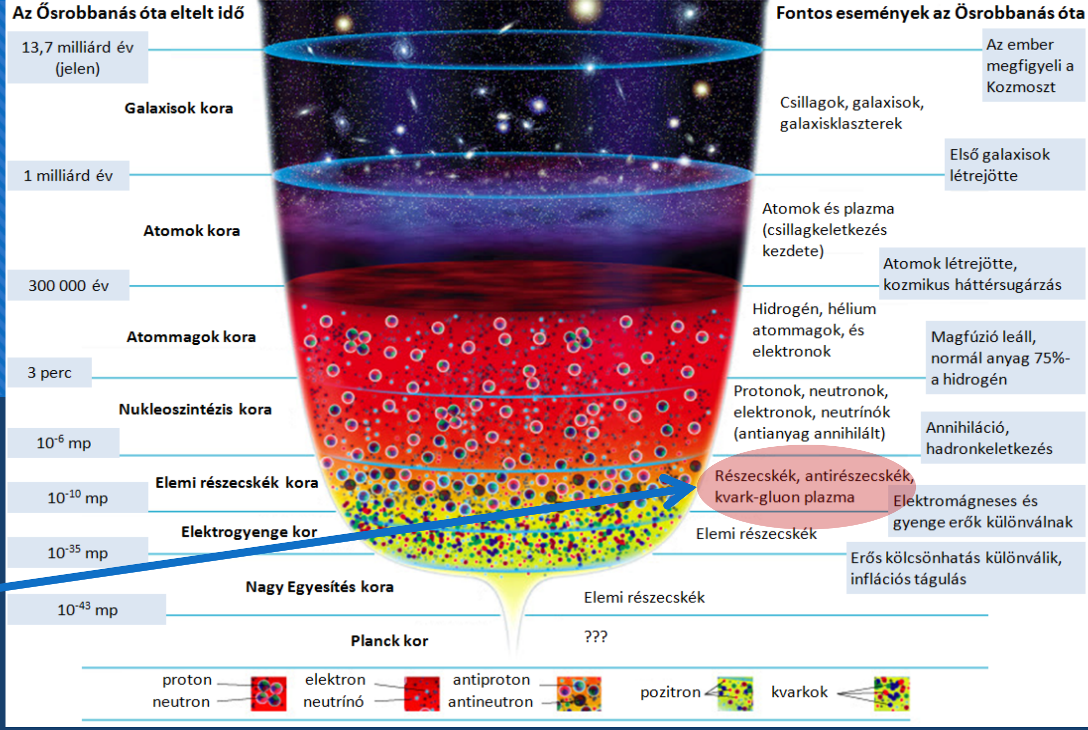


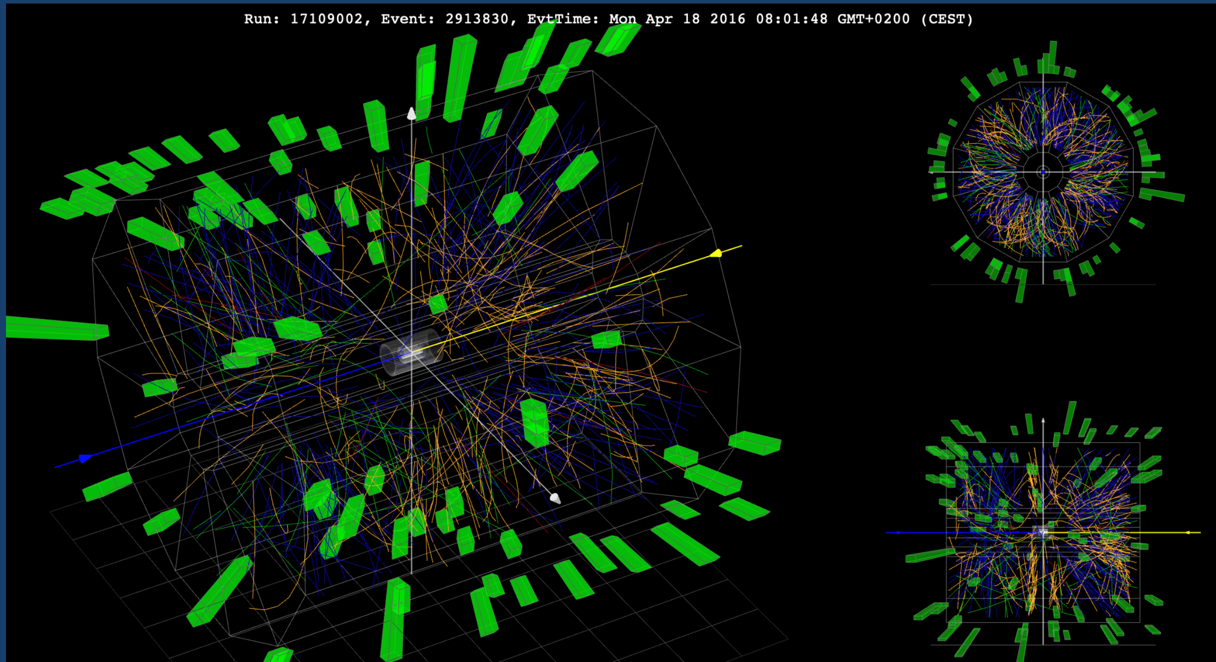
$C(q)$ correlation function

Measuring $C(q)$: map out source's space-time geometry on the femtometer scale!

Big bang in the lab

- Universe: ~13,7 billion years old
- ~10^{-6} s: quark-gluon plasma
- Collisions at ultrarelativistic velocities: „small bang”
- Detectors around the collisions: reconstruction of the state right after the collision

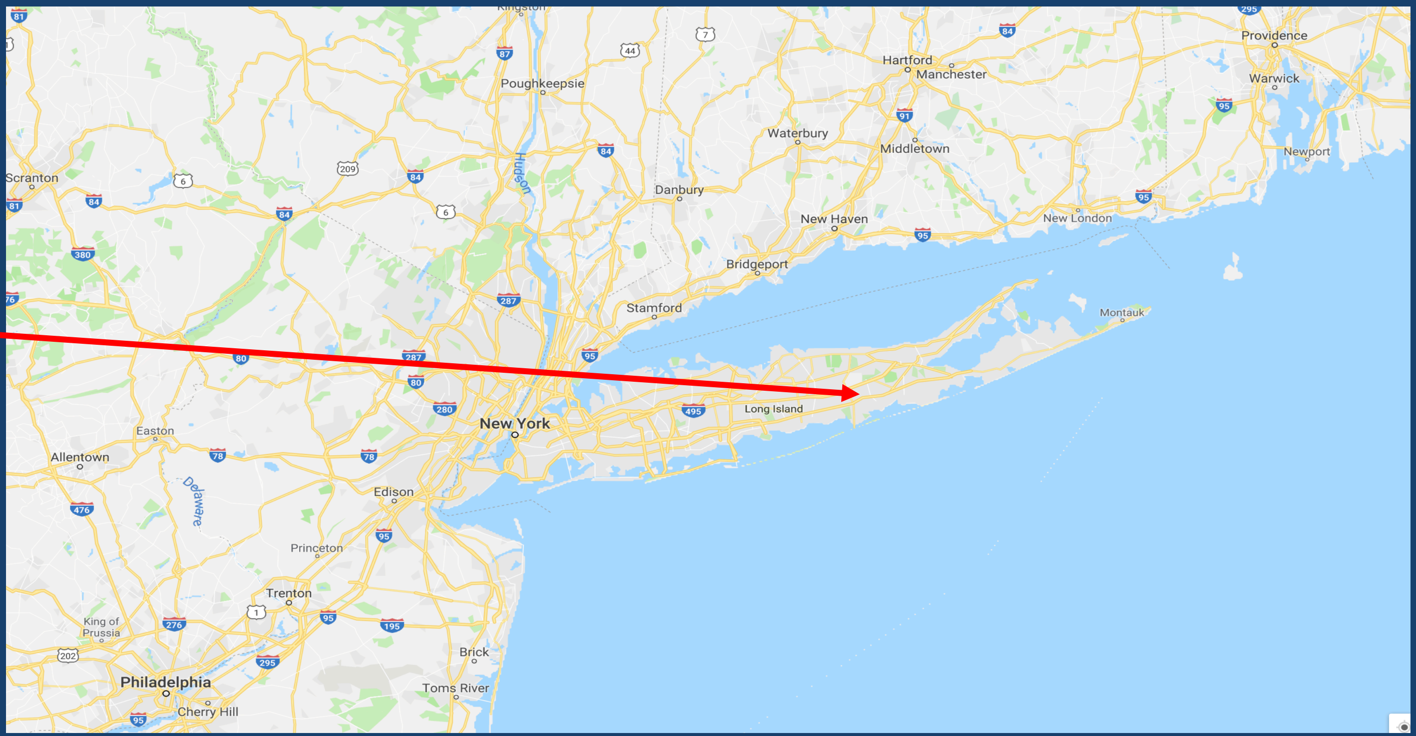




Relativistic heavy ion collider (RHIC)

Aerial photo of RHIC

Long Island, USA



Femtoscopy in HEP

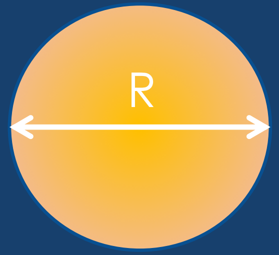
Notations:

-$Q = |\vec{q}_{LCMS}|$: invariant to Lorentz-boost in the z direction

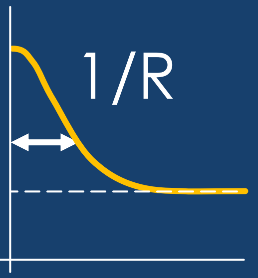
-$Q = \sqrt{(p_{1x}-p_{2x})^2 + (p_{1y}-p_{2y})^2 + q^2_{long,LCMS}}$ where

$$q^2_{long,LCMS} = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1+E_2)^2 - (p_{1z}+p_{2z})^2}$$

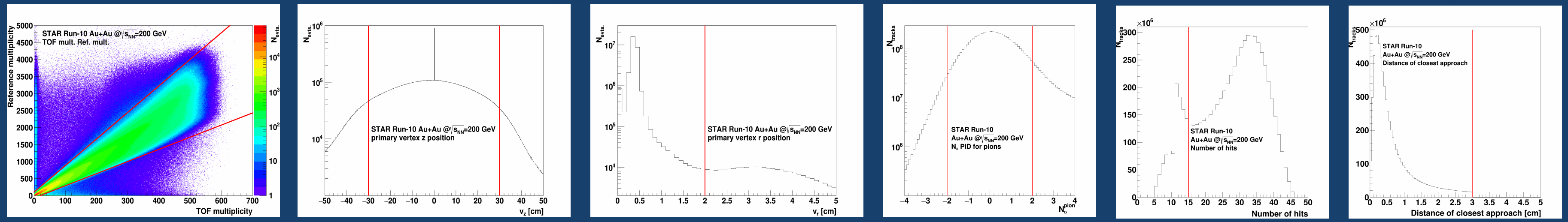
- If S(r) symmetric Lévy: $C(Q) = 1 + \lambda \cdot e^{-(RQ)^{\alpha}}$ (without corrections)
  - λ: **correlation strength**
  - R: **source parameter**
  - α: **Lévy-exponent**
- Previously Gauss shape: $\alpha = 2$, not precise
- Lévy-type correlation function: $\alpha \neq$ fixed
- I measured: $C(Q)$ for charged pion pairs @ 200 GeV @ STAR experiment
- Collision data needs the application of cuts (event- ,particle- , pair-cuts)

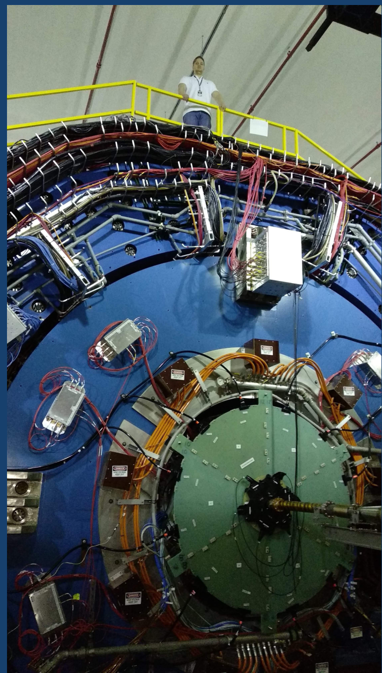


$S(r)$

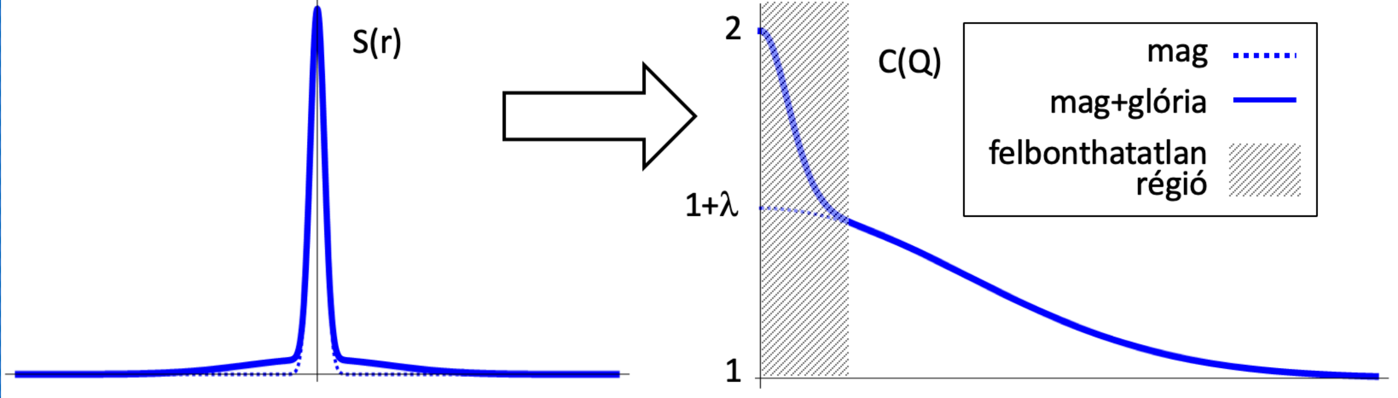


$C(Q) = 1 + |\tilde{S}(Q)|^2$





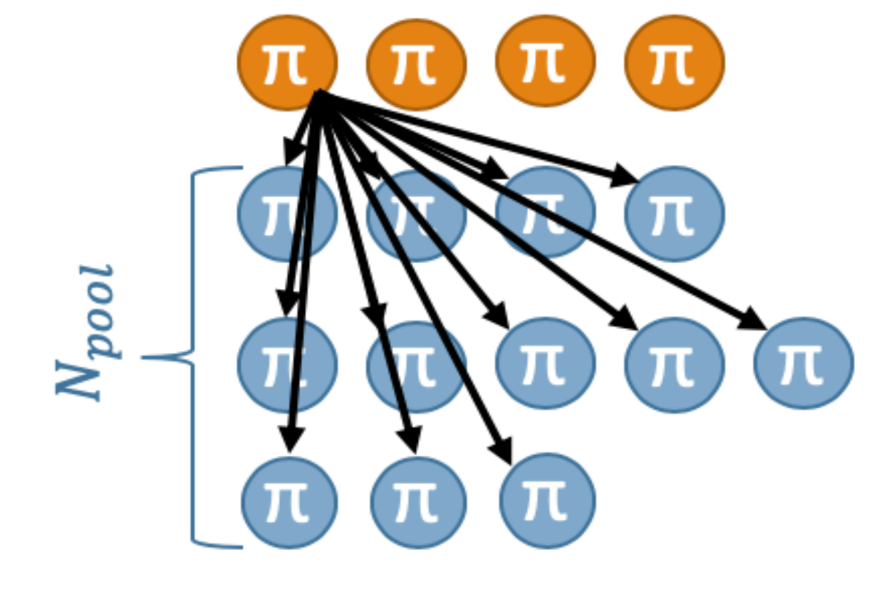
Core-halo model

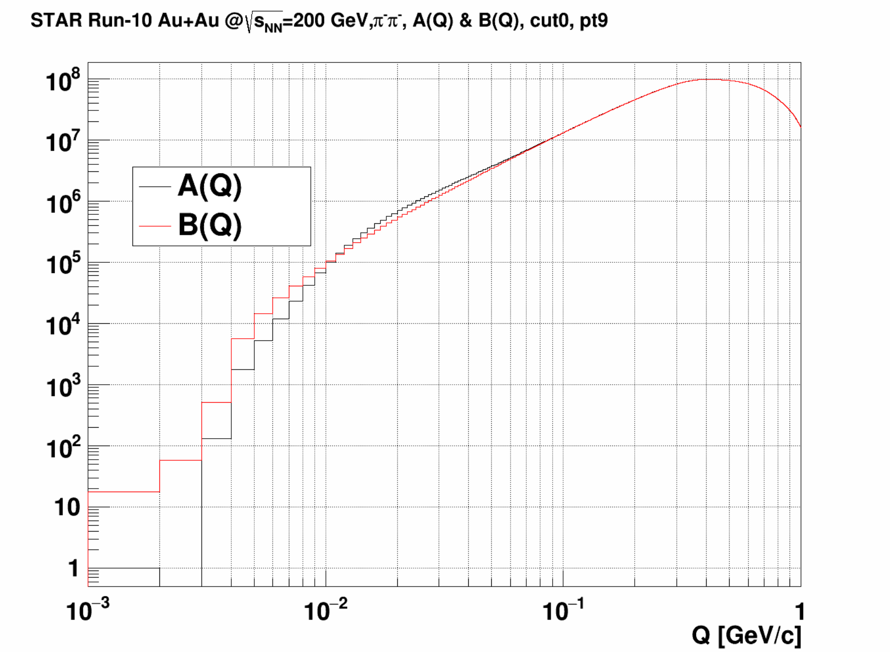


- $C_2(q,K) = 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}$ → if we neglect final state interactions: $C_2(Q=0) = 2$
- However, we cannot measure C_2 at rel. mom. = 0, only at a small finite Q_{min}, then we extrapolate to $Q = 0$
- Due to the extrapolation, the extrapolated value can differ from the real value, which we can take into account as: $\lambda = \lim_{Q \to 0} C_2(Q) - 1$ ← correlation strength
- By measuring collison data, $\lambda < 1$ could be seen → **core-halo model**
- Not all particles are coming from the collision, several comes from decays
- Core-halo model: the source function consists of core & halo parts
- After long calculations, we can get: $\sqrt{\lambda} = \frac{N_{core}}{N_{core} + N_{halo}}$

Event mixing

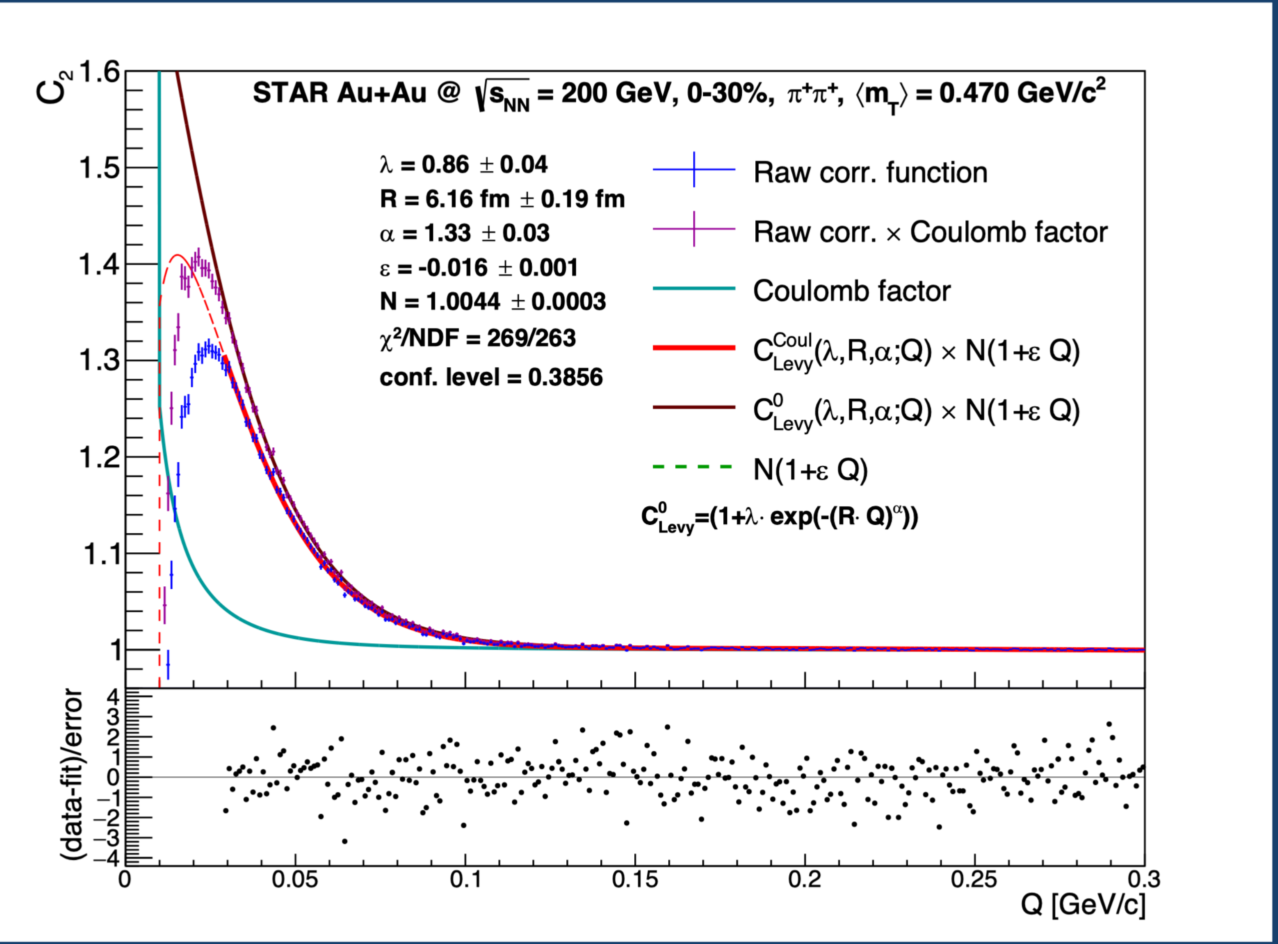
- **Measure: relative momentum distribution of pion pairs**
- **A(Q)**: 2 pions from the same event (**actual distr.**)
- **B(Q)**: 2 pions from different events (**background distr.**)
- B(Q) and A(Q): same kinematic and acceptance conditions
  - Centrality and vertex position must be similar
  - Applying: 5% wide centrality, 2 cm wide z-vertex bins
- $C_2(Q) = A(Q)/B(Q)$ (needs to be normed)





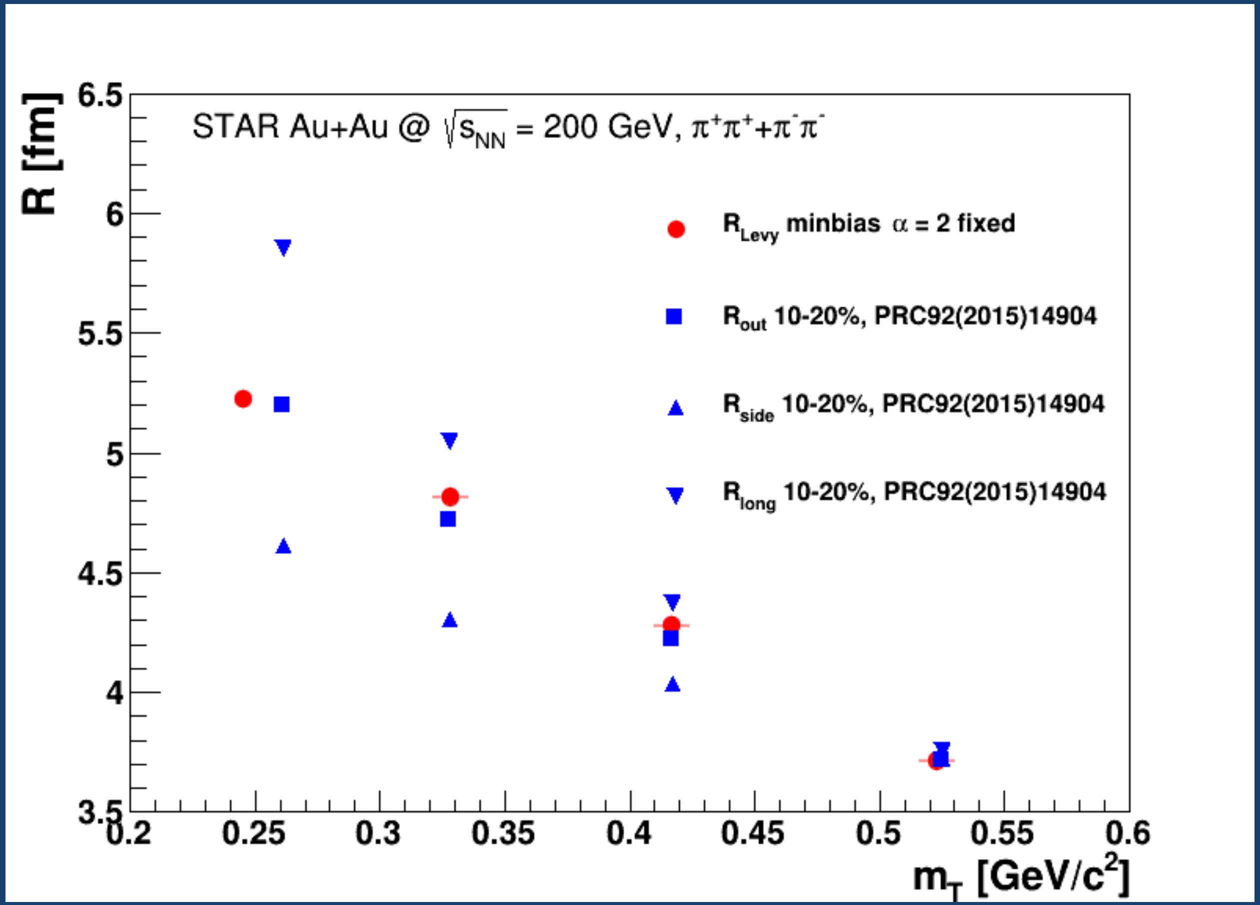
Fitting the measured correlation functions

- ROOT package (C++), MINUIT2 χ^2 minimizer
- 4 and 30 m_T (pair transverse mass) bins: [212,792] MeV/c² ($m_T = \sqrt{m^2 + K_T^2/c^2}$)
- Fitting parameters:
  - λ, R, α : **physics parameters**
  - $N \approx 1$: **normalising patameter**
  - $\varepsilon \approx 0$: **linear background parameter**



Comparison: STAR

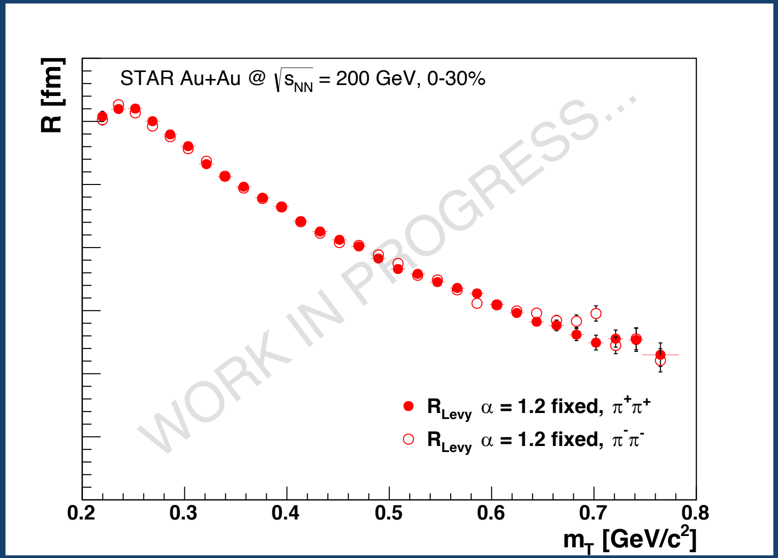
- First step: comparing with previous results
- STAR HBT: Phys. Rev. C92 (2015) 14904
    - Gauss shape fitting: $\boldsymbol{\alpha = 2}$
    - 4 m_T bin
    - λ parameter is not published
    - **3 dimensional measurement**
    - *Trend and average value of R is comparable*
- My measurement is compatible with the previously published

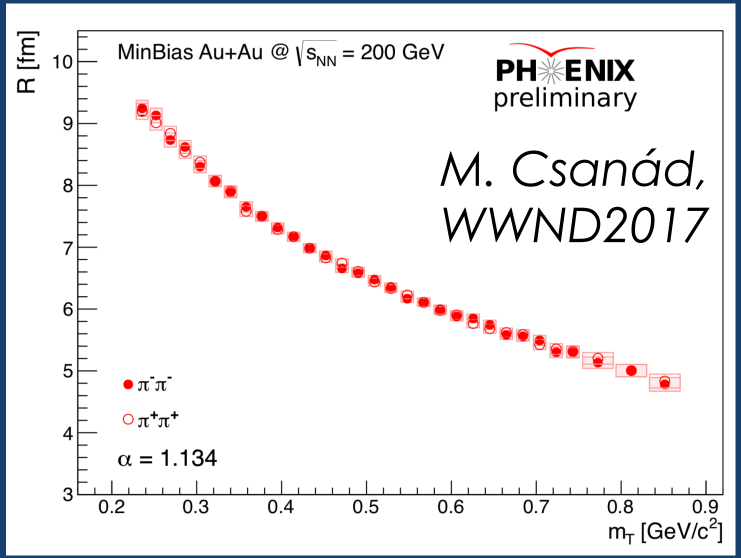


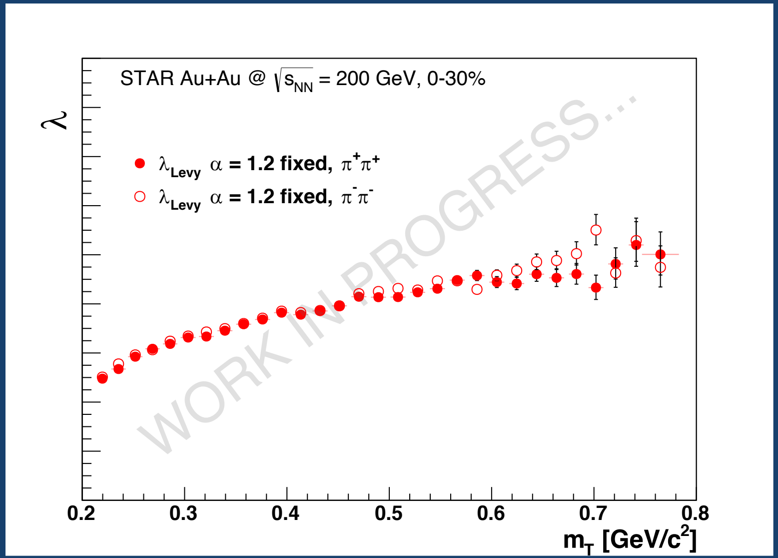
Comparison: PHENIX fixed α

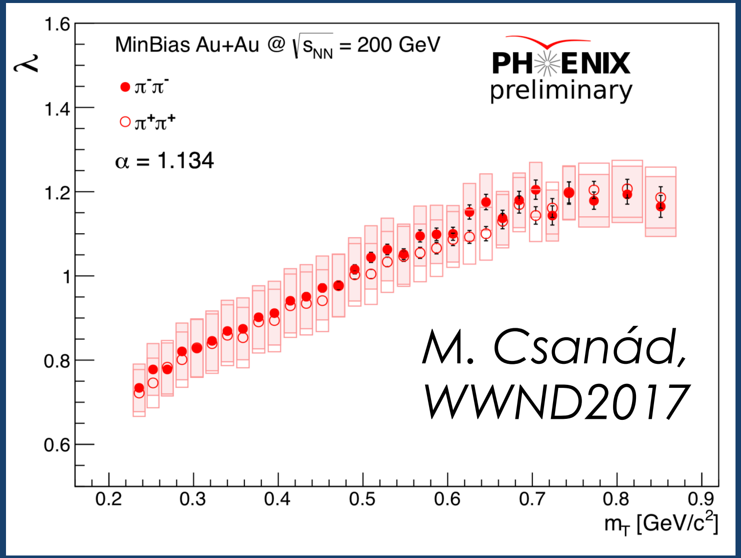
- $\alpha = 1.2$ fixed
- $30\ m_T$ bins
- My results reproduce the PHENIX experiment's published results

Hivatkozás: M. Csanád, WWND2017



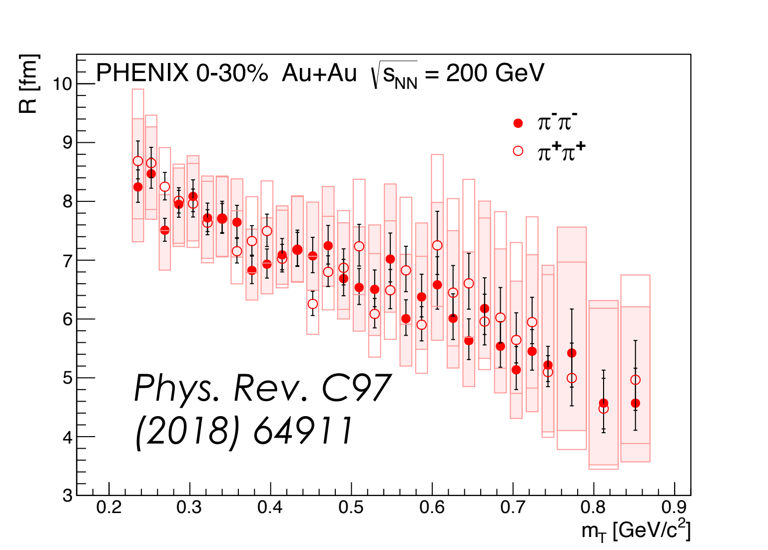


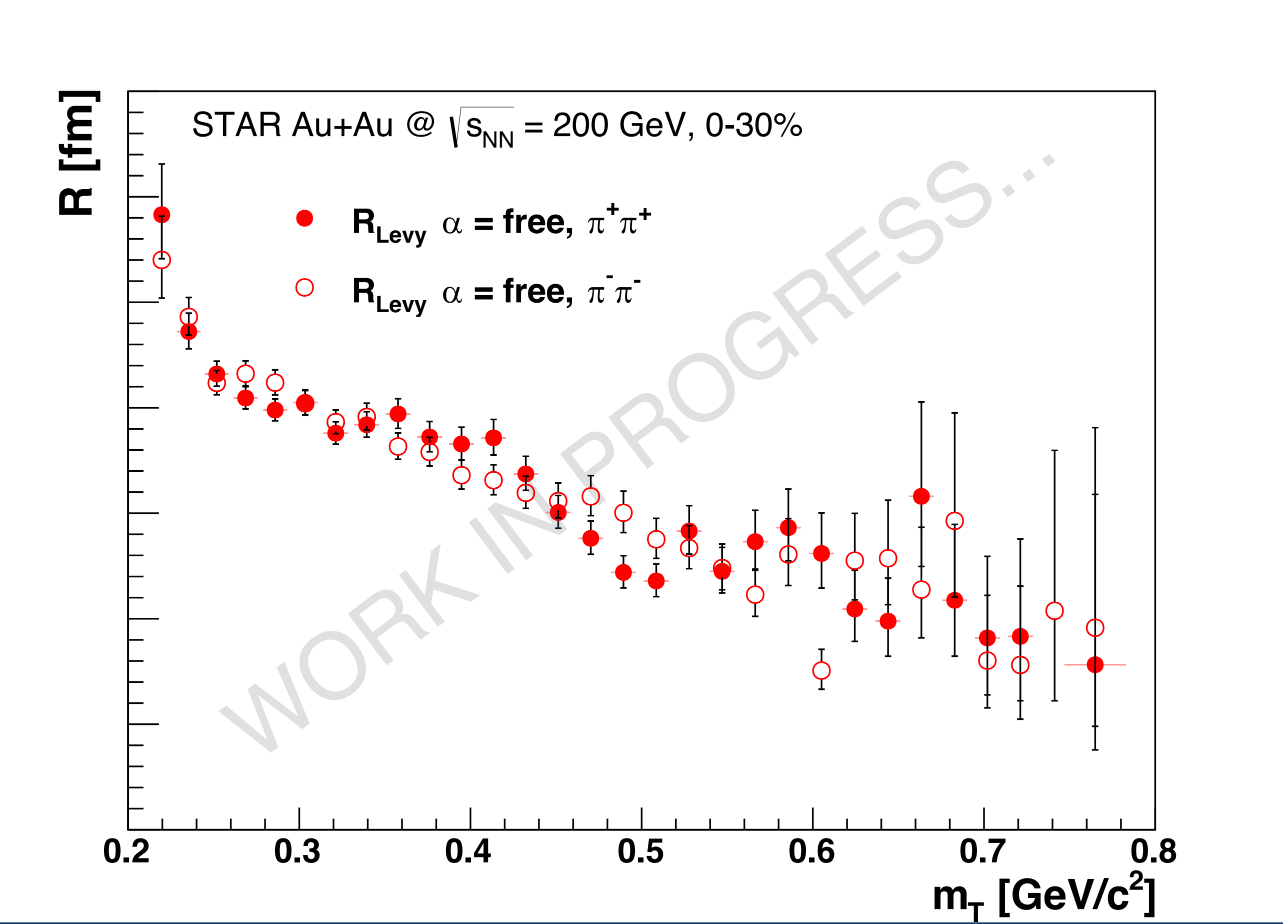




Fitting with $\alpha \neq$ fixed: R physics parameter

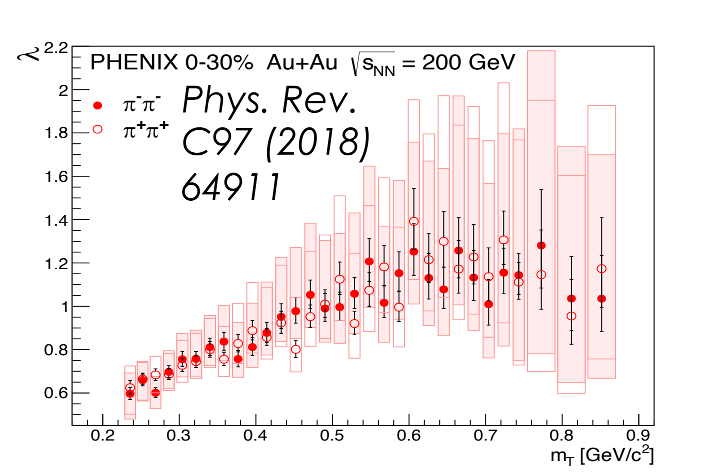
- Still decreasing trend
- Corresponds to the hydrodinamical calculations' radial expansion $R^2 \sim 1/m_T$ prediction
- PHENIX comparison: Phys. Rev. C97 (2018) 64911

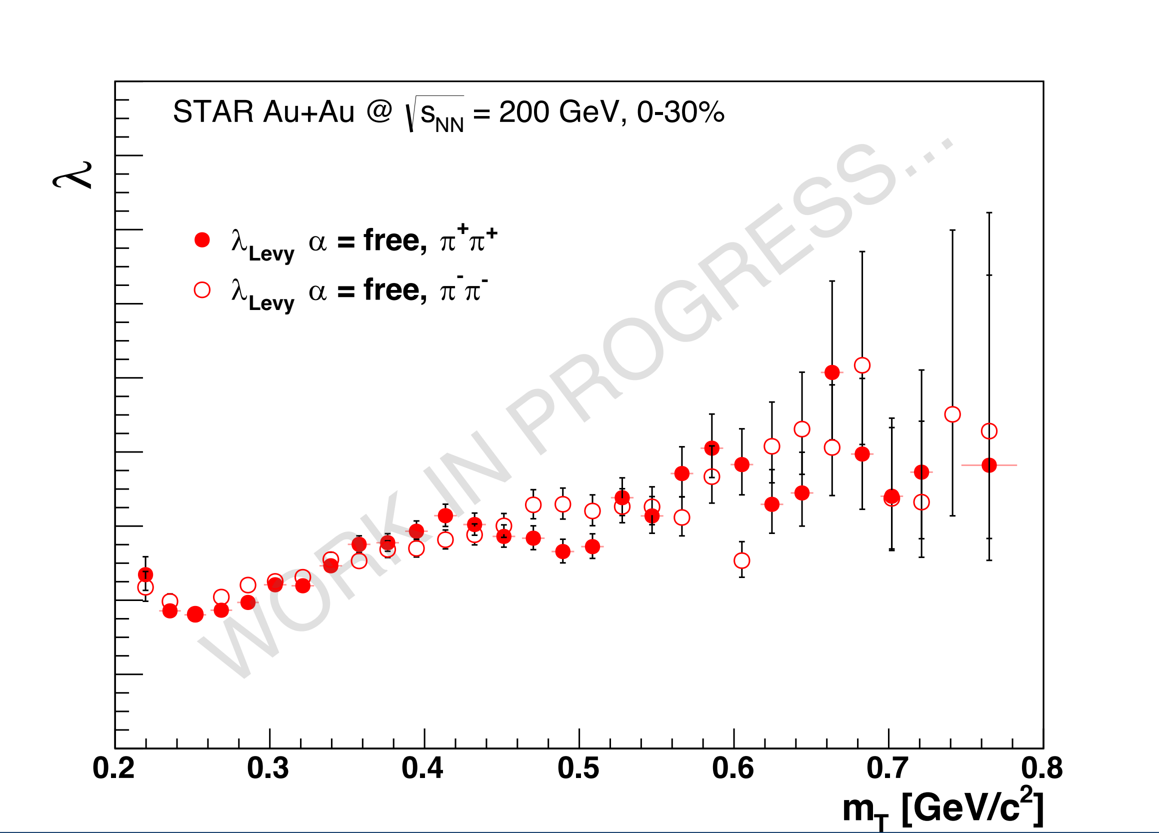




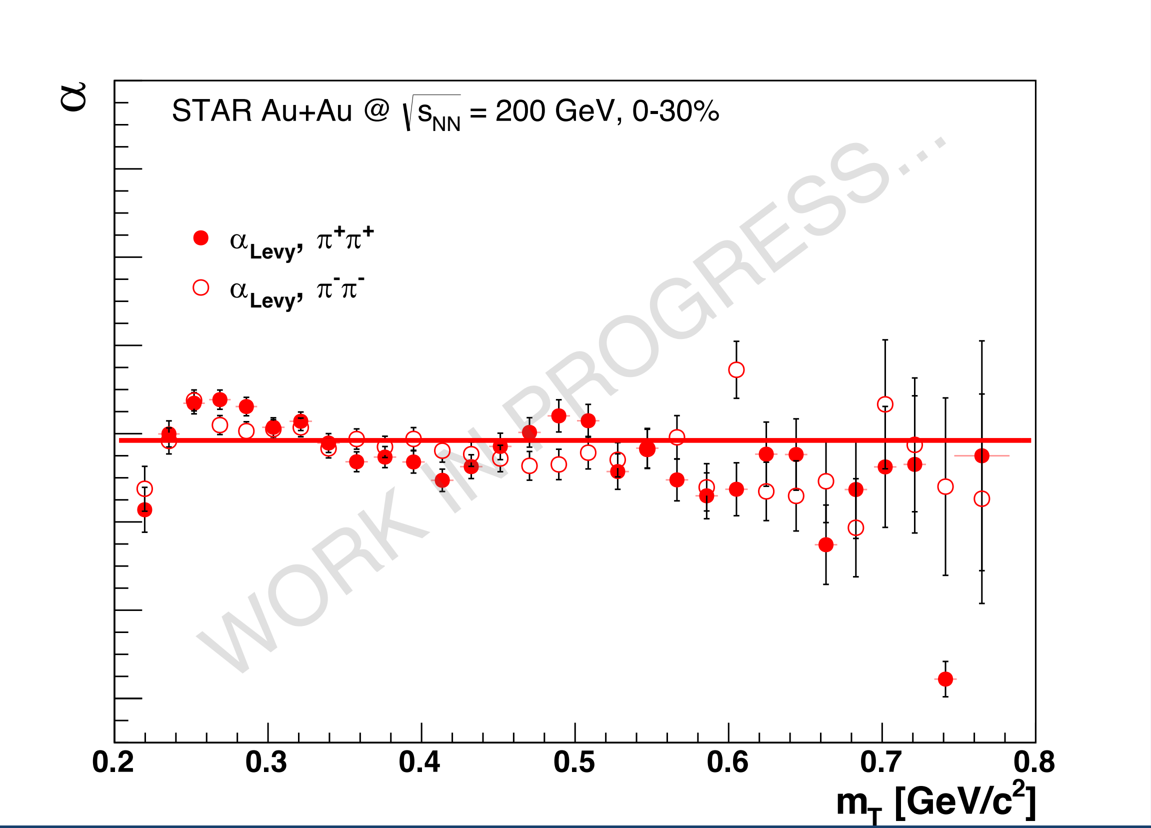
Fitting with $\alpha \neq$ fixed: λ physics parameter

- $\lambda(m_T) \neq 1$ could have different reasons
  - Smaller m_T: numerous pions are created from long lifetime resonance (such as η' meson) decays
- Bigger m_T: λ saturates, smaller m_T: λ is smaller
- PHENIX comparison: Phys. Rev. C97 (2018) 64911



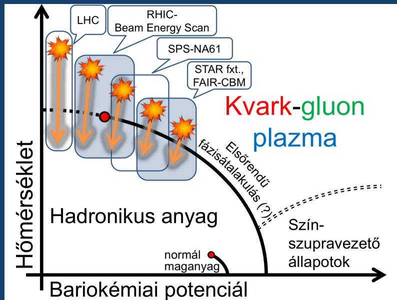


Fitting with $\alpha \neq$ fixed: α physics parameter



- α is nearly m_T independent, $\langle\alpha\rangle \approx 1.2$
- $\alpha <\ 2$: Gauss shape is not correct
- Relationship with the critical point:
  - Critical correlation exponent η, universality
  - Meaning: spatial correlation's power-low tail (hatványlecsengés)
  - Same with the Lévy-exponent
    Ref.: Csörgő, et al, AIP Conf. Proc. 828 (2006) 525-532, nucl-th/0512060
  - Random field 3D Ising-model: $\eta \simeq 0.5 \pm 0.05$
    Ref.: *H. Rieger, 1995. Phys.Rev.,B52 (1995) 6659*
  - 3D Ising-model: $\eta \simeq 0.03631$
    Ref.: *S. El-Showk et al., J.Statist.Phys. 157 (2014) 869*
- It seems important to measure in smaller enegy data ⇒ STAR Beam Energy Scan!

Quark-hadron transition:
big E: continous
small E: first-order phase transition

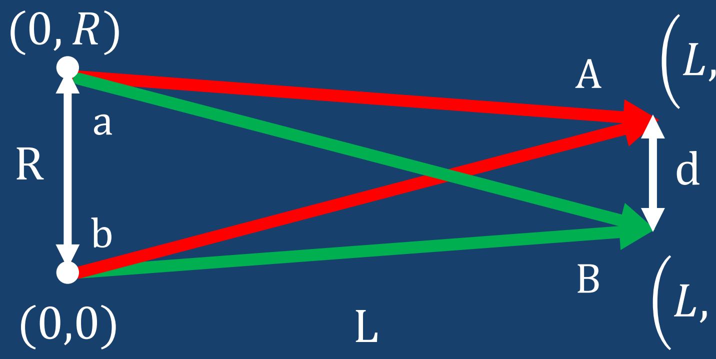


Conclusions

- HBT-effect and Bose—Einstein-correlations were introduced
- RHIC STAR: 2010, Au+Au $\sqrt{s_{NN}} = 200$ GeV
- Gauss shape is not correct ⇒ Lévy-distribution
- Lévy-distribution + final state Coulomb-interaction ⇒ the data can be described statistically well
- Fitting: Lévy source parameters' (R, λ, α) m_T dependence
- α Lévy-exponent < 2, but far away from the critical point's expected $\alpha < 0.5$ value
- α looks independent from m_T
- High energy heavy ion collisions: correlation functions' Lévy-exponent is **measurable**
- Plans: 3D measurements with differently stored data and with improved fitting for my MSc thesis

Thanks for your kind attention!

Back-up



$$\left(L, \frac{R+d}{2}\right)$$

$$\left(L, \frac{R-d}{2}\right)$$

$$\vec{r}_{aA} = \left(L, \frac{-R-d}{2}\right)$$

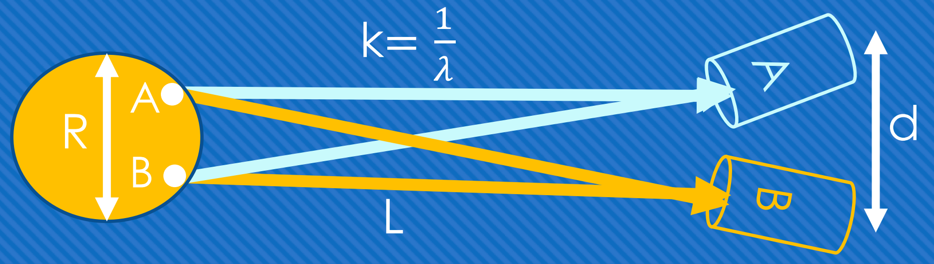
$$\vec{r}_{aB} = \left(L, \frac{-R+d}{2}\right)$$

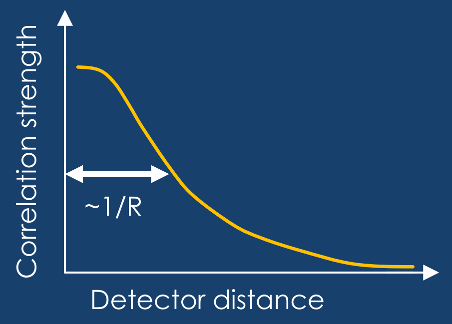
$$\vec{r}_{bA} = \left(L, \frac{+R+d}{2}\right)$$

$$\vec{r}_{bB} = \left(L, \frac{+R-d}{2}\right)$$

$$r_{aA} - r_{bA} + r_{aB} - r_{bB} = 2\sqrt{L^2 + \frac{(R+d)^2}{4}} - 2\sqrt{L^2 + \frac{(R-d)^2}{4}} \approx \frac{Rd}{L}$$

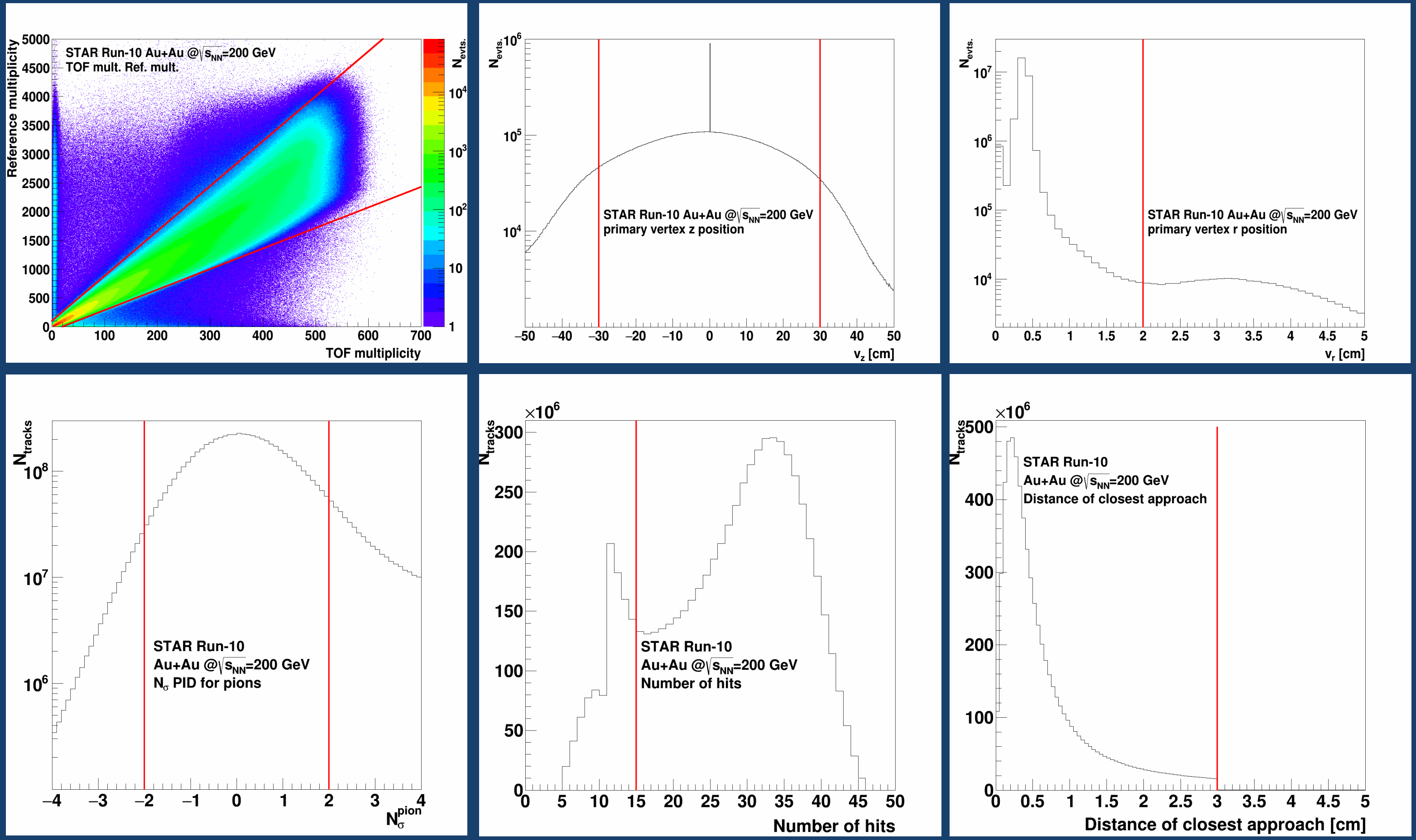
$$k(r_{aA} - r_{bA} + r_{aB} - r_{bB}) \approx \frac{kRd}{L} = R \cdot \frac{kd}{L} = R \cdot \Delta k$$


R
A
B
k= 1/λ
L
A
B
d


Correlation strength
~1/R
Detector distance

Mérés részletei

- **Eseményválogatás**:
  - TOF és TPC multiplicitása ~ arányos
  - Ütközés (vertex) helye: $|v_z| < 30$ cm, $v_r < 2$ cm
- **Részecskeválogatás**:
  - Részecskeazonosítás: 2σ
  - TPC beütések száma > 15
  - Nyom-vertex távolság (DCA) < 3 cm
- **Párválogatás**:
  - Nyomok összeolvadása
  - Nyomok szétválása



Általam elemzett ütközési adatok

BES I.

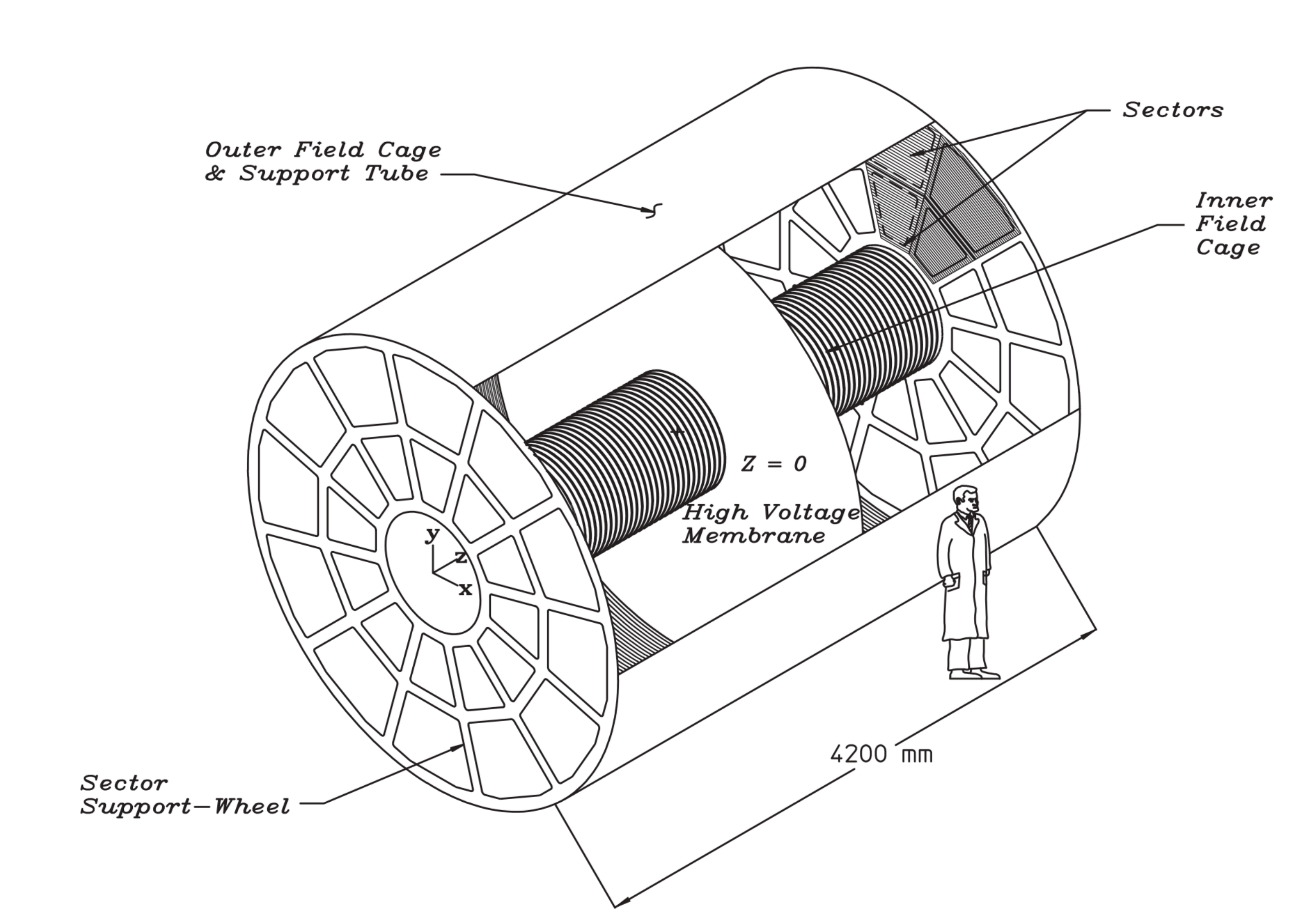
Tervek: BES II.

$\sqrt{s_{NN}}$ [GeV]	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
510																		p+p		
500										p+p		p+p	p+p	p+p						
400						p+p														
200		Au+Au	p+p	d+Au, p+p	Au+Au, p+p	Cu+Cu, p+p	p+p	Au+Au	d+Au, p+p	p+p	Au+Au	Au+Au	p+p, Cu+Au		He+Au, Au+Au	p+p, p+Al, p+Au	Au+Au, d+Au		Ru+Ru, Zr+Zr	
193													U+U							
130	Au+Au																			
62.4					Au+Au	Cu+Cu	p+p				Au+Au						d+Au			Au+Au fix target
54																		Au+Au	Au+Au fix target	
39											Au+Au						d+Au			
27												Au+Au							Au+Au	
22.5		Au+Au				Cu+Cu														
19.6												Au+Au					d+Au			Au+Au fix target, Au+Au
14.6															Au+Au	Au+Au				Au+Au fix target, Au+Au
11.5											Au+Au									
9								Au+Au	Au+Au											
7.7											Au+Au								Au+Au fix target	
5													Au+Au							

p+p, Au+Au, Au+Au fix target, d+Au, Cu+Cu, U+U, Cu+Au, He+Au, p+Au, p+Al, Zr+Zr, Ru+Ru

TPC

- Track rekonstrukció, p, dE/dx (ionizáció)
- STAR fő nyomkövető detektora
- 4.2 m hosszú, 4 m széles gázkamra, közel egyenletes 135 V/cm elektromos térben
- Gázkamrában haladó töltött részecske: ionizálódik, elektronkibocsátás, becsapódás
- Kiolvasó rendszer: sokszálas proprocionális karma elven működik
- 0.5 T nagyságú szolenoid mágneses mezejében van
- P10 gázzal van töltve: 10% metán, 90% argon
- 2 mbar nyomással a légköri nyomás fölött

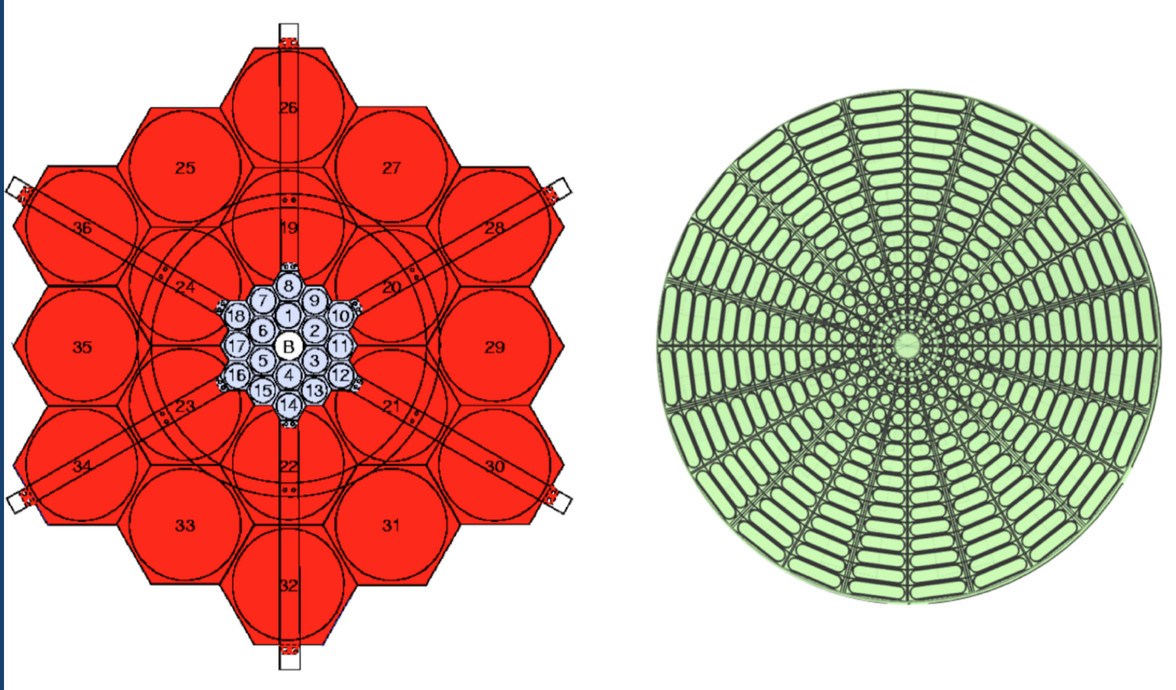


TOF

- Repülésidő-detektor
- TPC külső részén helyezkedik el
- 100 ns-os időfelbontás, nagy (>95%) detektálási hatékonyság
- TPC-vel együtt a részecskeazonosítást segíti
- Mit mér? – részecskék sebességét
- TPC: impulzust mér
- Pályarekonstrukcióból megkapható a track megtett L úthossza
- Sebesség + impulzus + úthossz = tömegnégyzet meghatározható

BBC

- Beam-Beam Counter
- Két szcintillációs detektorból áll, 3.75 méterre a TPC közepétől
- Mindkét BBC 2-2 külső és belső gyűrűből
- Gyors válaszidejű detektor, ezért események triggerezésére tökéletesen alkalmazható, főleg alacsonyabb nyalábenergiákon
- Triggerezés = előírt tulajdonságoknak megfelelő eseményeket rögzítünk
- Időkülönbség a két BBC detektor között: lehetőség a z vertex mérésére



(a) BBC (b) EPD

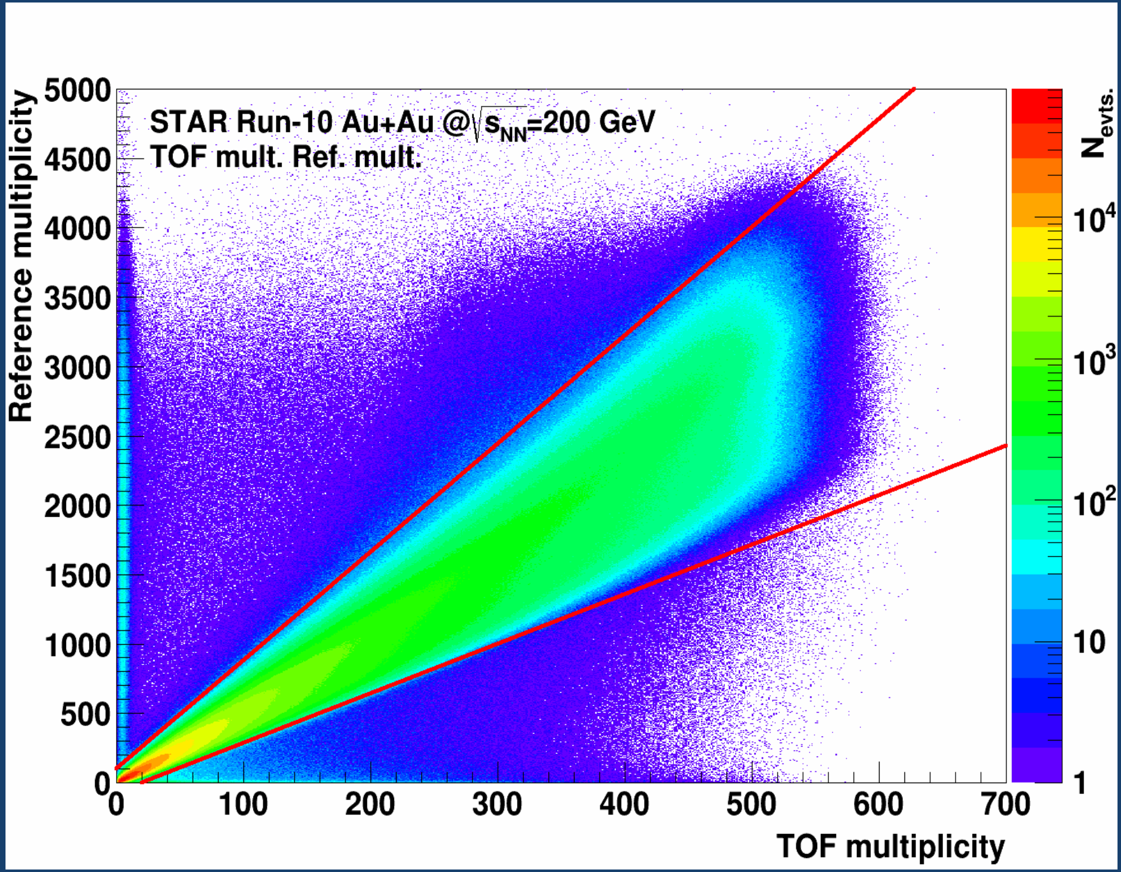
n-ed rendű eseménysík:

$$\psi_n \cdot n = \arctan\left(\frac{\sum_i w_i \sin(n\phi_n)}{\sum_i w_i \cos(n\phi_n)}\right)$$

w_i: súlyozási faktor
ϕ_n: azimut szög

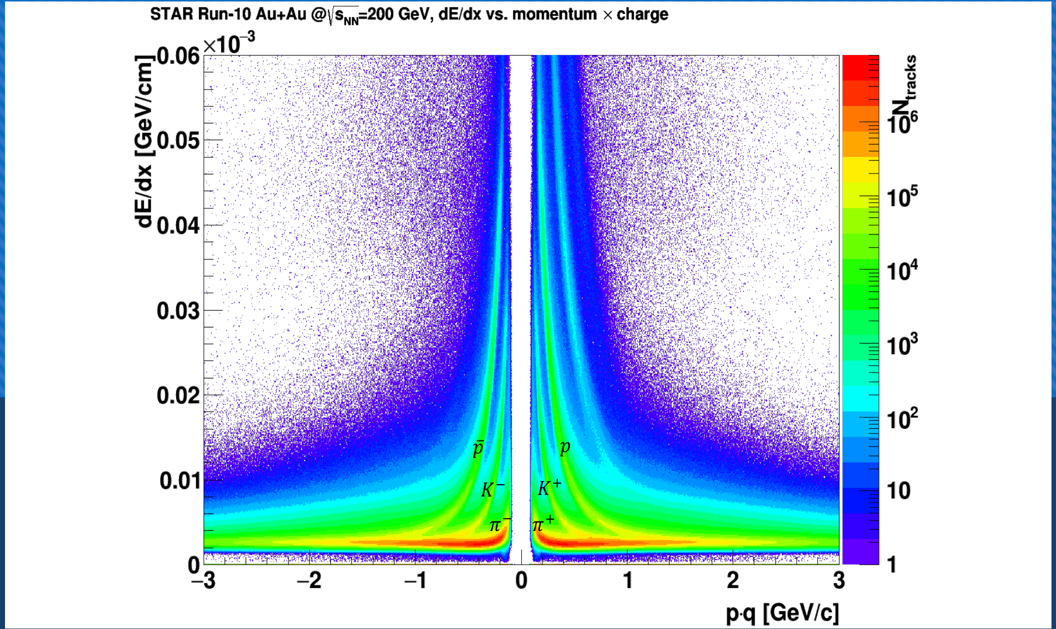
Eseményválogatás

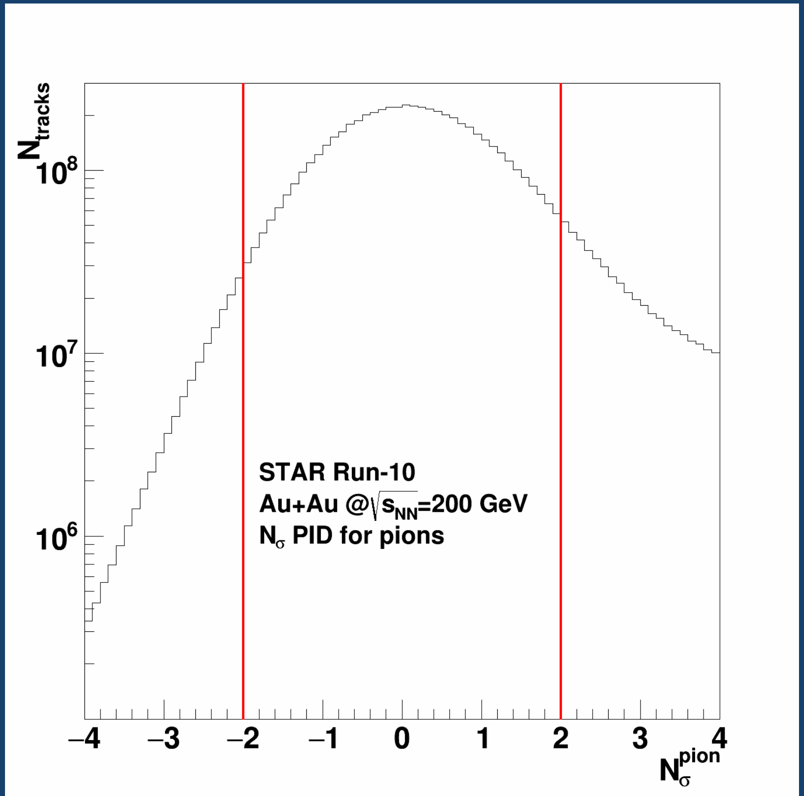
- TOF és TPC multiplicitása ~ arányos :
  - Multiplicitás: egy eseményben hány pion van
  - A tengelyek mentén megfigyelhető események esetén valószínűleg nem volt minden rendben, vágással eldobjuk azokat az eseményeket



Részecskeválogatás

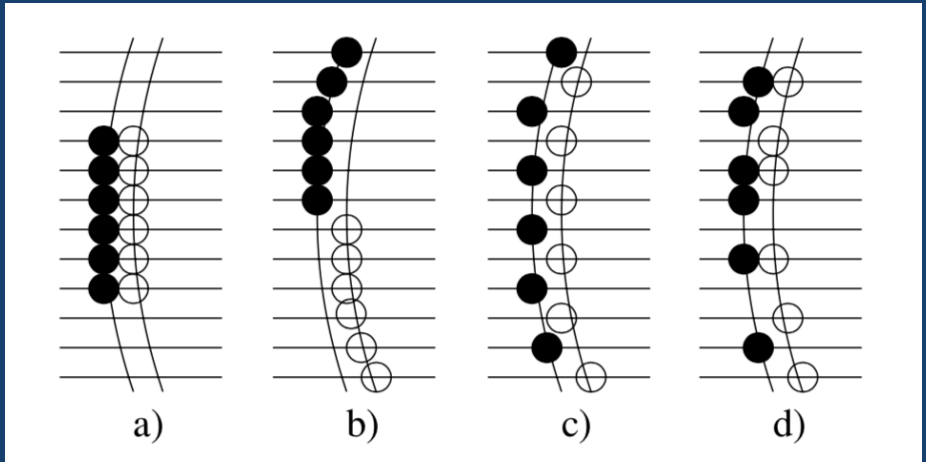
- Részecskeazonosítás: dE/dx
- dE/dx-ből: vetületet adott impulzus esetén, különböző csúcsok megjelennek
- Pionra vonatkozó részre normál eloszlás illesztés (Gauss)
- Normál eloszlás ⇒ 2σ biztosságon belül legyen pion, és kívűl legyen kaon, proton, elektron





Párválogatás

- Figyelembe kell venni: detektorok hatékonysága, részecskepálya rekonstrukciós algoritmusának jellemzői
- Algoritmus részecskepályát kettéoszthat: track splitting
- Algoritmus két különböző részecskepályát lehet, hogy nem tud megkülönböztetni: track merging
- Bonyolult technikai megvalósítás, STAR cikk alapján kezeltem őket: STAR Collaboration, J. Adams et al., Phys. Rev. C 71, 044906 (2005)
- Track splitting ábra:
  - (a) eset: tisztán két track
  - (b) eset: lehetséges split track
  - (c) eset: lehetséges split track
  - (d) eset: valószínűleg két track



Lévy illesztés, Coulomb-kölcsönhatás

- Lévy-típusú korrelációs függvény végállapoti kölcsönhatások nélkül: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- Sinyukov módszer: $C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot \underbrace{N \cdot (1 + \varepsilon Q)}$

Korreláció erőssége

Coulomb korrekció

R: Lévy skála paraméter
α: Lévy exponens

Lehetséges lineáris háttér (általában elhanyagolható)

- Coulomb korrekció: $K(Q;\alpha,R) = \dfrac{\int D(r)\left|\psi^{Coul}(r)\right|^2 dr}{\int D(r)\left|\psi^{(0)}(r)\right|^2 dr}$ → **Numerikusan számolva**

Térbeli korrelációk (pár-forrásfüggvény)

Kétrészecske hullámfüggvény

Lévy illesztés, Coulomb-kölcsönhatás

- Coulomb korrekció (numerikusan számolva):

$$D(r) = \mathcal{L}(\alpha, 2^{\frac{1}{\alpha}}R, r)$$

$$\psi^{Coul}(\boldsymbol{r}) = \frac{1}{\sqrt{2}}\frac{\Gamma(1+i\eta)}{e^{\pi\eta/2}}\{e^{i\boldsymbol{qr}}F\big(-i\eta, 1, i(qr-\boldsymbol{qr})\big) + e^{-i\boldsymbol{qr}}F(-i\eta, 1, i(qr+\boldsymbol{qr}))\}$$

$$K(Q;\alpha,R) = \frac{\int D(r)\left|\psi^{Coul}(r)\right|^2 dr}{\int D(r)\left|\psi^{(0)}(r)\right|^2 dr}$$

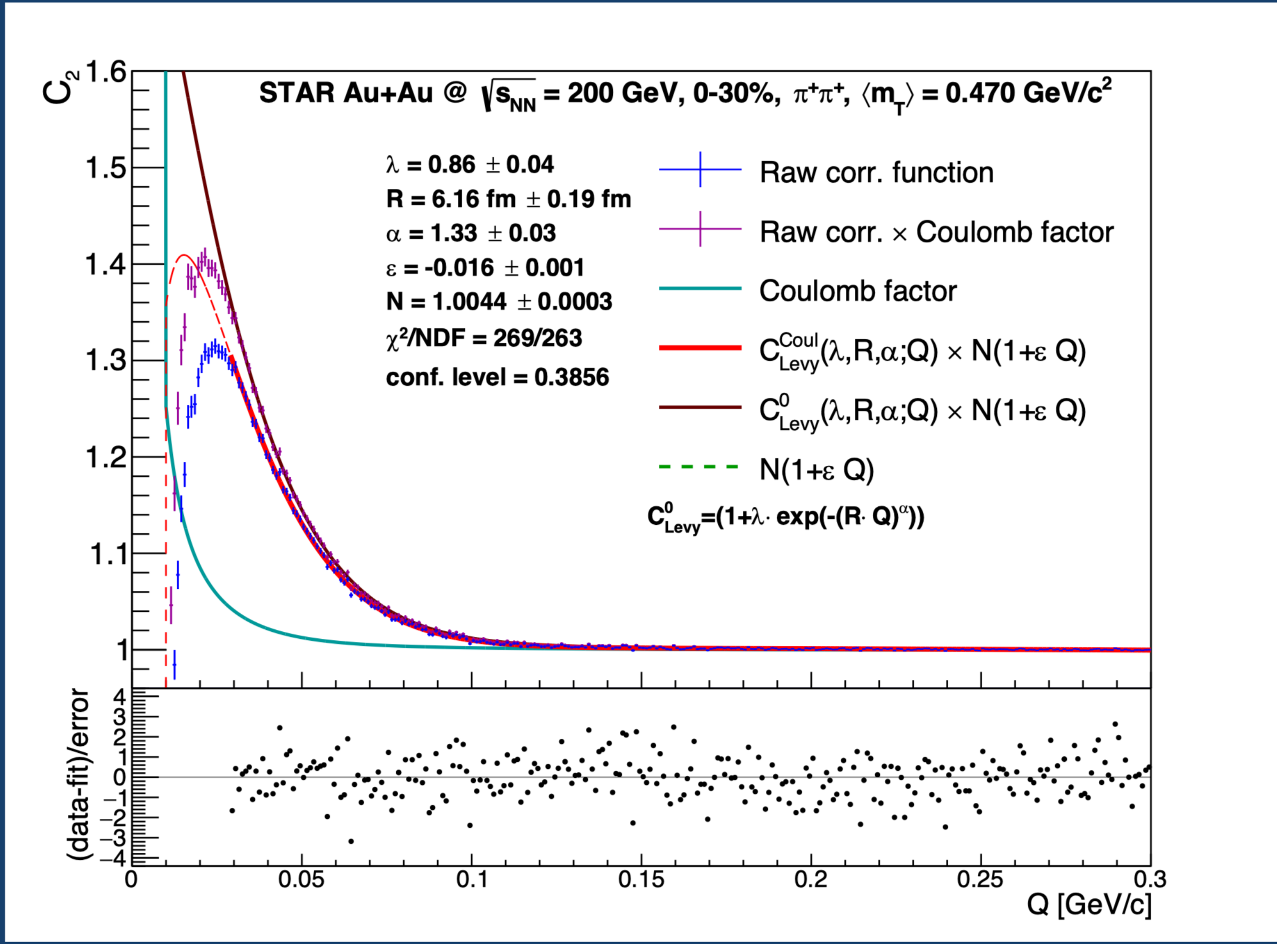
r: részecskepár térbeli szeparációja
q: 3D relatív impulzus a pár nyugalmi rendszerében (PCMS)
Γ: Gamma függvény
F(a,b,z): elfajult hipergeometrikus függvény

$$\eta = \frac{m_\pi c^2 \alpha}{2\hbar qc}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{\hbar c} \approx \frac{1}{137}$$

A korrelációs függvények illesztése

- Illesztési paraméterek:
  - λ, R, α : **fizikai paraméterek**
  - $N \approx 1$: **normálási paraméter**
  - $\varepsilon \approx 0$: **lineáris hátteret jellemző paraméter**
- $N \approx 1$ és $\varepsilon \approx 0$: lehetséges hosszútávú korrelációs hátteret reprezentálnak
- Ábramagyarázat:
  - Mért korrelációs hisztogram: kék adatpontok
  - Teljes illesztett függvény: piros görbe
  - Coulomb-korrigált függvény: barna görbe
  - Coulomb-korrigált adatpontok: lila adatpontok
  - Illesztés a folytonos vonallal jelölt tartományon történt



Ising-modell

- Statisztikus mechanikában ferromágnesesség matematikai modellje
- A modell diszkrét változókból áll, amik atomi spinek mágneses dipól momentumait reprezentálják
  - Két állapotban lehetnek: +1, -1
- A spinek rácsban vannak elhelyezve, kölcsönhathatnak a szomszéddal
- A modell lehetőséget nyújt fáziásátalakulások identifikálására
- Háromdimenziós Ising-modell:
  - Kritikus pontja kvantumtérelmélet által leírva, Monte Carlo szimulációkra és elméleti megfontolásokra támaszkodva
- Univerzalitás osztály: kritikus exponens értéke minden modellre ugyan annyi egy osztályban
- Kritikus exponens: fizikai tulajdonságok viselkedését leírja a folytonos fázisátalakulás körül