

WORKSHEET 5.

topic: Time dependence of radioactivity

1. What mathematical function can describe the radioactivity in time.

A = activity = number of decayed atoms per time

Time dependence of radioactivity

Radioactive equilibrium

exp. decay

$$A(t) = A_0 e^{-\lambda t}$$

λ - decay constant $\frac{1}{s}$

$$\frac{1}{s} \cdot s = 1$$

$T_{1/2} = \frac{\ln 2}{\lambda}$

$2 = e^{\ln 2}$

$$A(t) = A_0 e^{-\lambda t} = A_0 e^{-\ln 2 \cdot \frac{1}{T_{1/2}} t}$$

$$= A_0 e^{\ln 2 \cdot (-\frac{t}{T_{1/2}})} = A_0 2^{-\frac{t}{T_{1/2}}}$$

$A(t) =$
 $e^a = 1$
 $e^a = b$
 $a = \ln b$
 $e^{mb} = b$
 $2^a = b$
 $a = \log_2 b$
 $e^x = 2$
 $x = \ln 2$

$T_{1/2} \rightarrow \frac{1}{2} A_0 = A(T_{1/2}) = A_0 e^{-\lambda T_{1/2}} \rightarrow 2 = e^{\lambda T_{1/2}} \rightarrow \ln 2 = \lambda T_{1/2}$

concepts: half life, decay constant, logarithm, $e=2,71\dots$, $\ln(2) = 0,69\dots$,
exponential decay law

2. What is behind the exponential decay law?

The exponential decay law, describes a mathematical function that is a **solution of a differential equation**. That differential equation is established from that the decay probability per time (decay speed) is constant in time for 1 atomic nucleus (for 1 atom)

Differential equation of simple decay

$$\frac{\Delta A}{\Delta t} = \frac{\Delta m}{\Delta t} = \frac{m(\text{tom}) - m(\text{today})}{1 \text{ day}} = \frac{m_{\text{tom}} - m_{\text{today}}}{1 \text{ hour}}$$

activity $\frac{\Delta A}{\Delta t} = -\lambda A$

$\frac{\Delta m}{\Delta t} \rightarrow \frac{dm}{dt}$ derivative

$\frac{\Delta N}{\Delta t} = -\lambda N$

number

$\lambda N = A$

solve \rightarrow find $f(t)$

$(e^{-\lambda t})' = -\lambda e^{-\lambda t}$

$\frac{dN(t)}{dt} = -\lambda N(t)$

$N(t) = N_0 e^{-\lambda t}$

$f'(t) = -\lambda f(t)$

$(e^x)' = e^x$
 $(e^{\lambda t})' = \lambda e^{\lambda t}$

$\frac{dN}{dt} = -\lambda N$

3. There is a very important formula that gives the relationship between the activity of a sample and the number of radioactive isotopes in the sample. **Why is it true?**

$$A = \lambda N$$

truth or not

definition of activity $\textcircled{1} A = \frac{N_{\text{decays}}}{T} = -\frac{\Delta N}{\Delta t}$

$$\Delta t \rightarrow 0 \quad -A = \frac{dN}{dt} = (N(t))' = (N_0 e^{-\lambda t})'$$

$$= -\lambda \underbrace{N_0 e^{-\lambda t}}_{N(t)} = -\lambda \cdot N$$

$$\textcircled{2} -A = \frac{dN}{dt} = -\lambda N \Rightarrow -A = -\lambda N$$

It can be mathematically derived from the exponential decay law.