1 Introduction

Every day life in our modern world is based on countless fields of physics that study the nuclei. The application of radioactive isotopes is extensive and there is a long list of industrial, medical, or environmental fields where their use is absolutely necessary.

The requisite for working with radioactive sources is that we are familiar with the properties of the decaying atoms: to know which radiations come from the nuclei or from the sample. It is important to know the activity of the source. Without the knowledge of the application methods we do not know the risk of usage these sources.

The aim of this laboratory practice is to give an introduction of the basics in working with the most used radioactive sources (\(^{60}\)Co, \(^{137}\)Cs and \(^{22}\)Na) and to be familiar with the decay of the nuclei in these preparations and to discuss the properties of these radiations. The experimental study of the activity of the selected sources is an occasion
to meet with the most elementary nuclear measuring techniques and to give an insights to the basic methods of evaluation of the nuclear physics measurements.

In the practice some concepts are probably occurred in your earlier studies (maybe in other contexts). These concepts are: activity, decay constant, half-life, annihilation, detection efficiency, $\gamma$-radiation, photoelectric effect, Compton-effect, pair production, mechanism of scintillation, photo-multiplier, differential discriminator.

2 Determination of the activity of radioactive sources

2.1 Decay scheme of nuclei

The definition of activity is applied to all kinds of decay. But the decay of the radioactive isotopes can be very diverse. The radioactive decay can happen in three different ways (alpha, beta and gamma decay). In the case of alpha decay the mass number of the nucleus reduces by four and the atomic number reduces by two. In the case of the beta decay the mass number is not changed but the atomic number increases by one ($\beta^-$ decay) or reduces by one ($\beta^+$ decay or K-capture or with other words: electron capture). In the case of gamma decay nor the mass number neither the atomic number changes.

In most cases the radiations after the transformations of the nuclei are mixed because in a radioactive decay more particles are emitted (one particle after another but in practical measurement this means in the same time). So an alpha or beta decay is usually accompanied by one or more emission of gamma photons. In case of positron decay after the positron slows down and meets an electron it will annihilate with it, due to this the rest of the energy of the two particles become radiation energy. With high probability two photons are created with 511 keV energy. With low probability – a few thousandths percent – it could happen that three photons share the released energy of 1022 keV. Also it might occurs that the decays do not always proceed in the same way, but in some cases the one kind of decay happens in other cases the other. These can be considered as the rival processes. Owing to this knowing the activity is not enough to characterize the number of the particles in a unit time that are emitted from the source. The decay scheme is necessary to be known.

In the Fig.1 there are three schemes of nuclei that are used in labor practice. These are the $^{60}$Co, the $^{137}$Cs and the $^{22}$Na. The most important attributes of the decay schemes can be recognized in Fig.1 the system of the excited states of each nucleus are presented with horizontal lines over the ground states. Next to each line, the differences among the energy of the excited states and the ground state is presented (its spin, parity, lifetime, etc.). The gamma decays that are actually happen are noted by vertical
Figure 1: The decay scheme of the isotopes that are used in the labor practice. Next to the energy levels there are the excitation energies compared to the ground state, on the left hand side there are the spin and the parity. The horizontal arrows present the gamma decays and the cross arrows present the beta decays where the atomic number change.
arrows. If the decay leads to another nucleus – except the gamma all kind of decay does – on the scheme of niveau or energy level the states of the parent element and states of the decay product are noted, which have a role in the decay scheme. The alpha, beta and gamma decays among the elements are noted by cross arrows. The half-life of the unstable isotopes is presented next to the proper line of the state. The percent value next to the arrows means the percentage of proper decay.

The positron decay is also noted by a cross arrow in the figure of the decay scheme of the $^{22}\text{Na}$. The positron decay are accompanied with other phenomenons, which cannot be in the figure. The positrons, which are electrically charged particle with large velocity, emitted from the radioactive nucleus and ionize in the matter that surrounds the sample while they lose energy and slow down. After they have slowed down, they eventually meet with an electron with similar momentum usually to create at least two photons with 511 keV energy, which photons move in the opposite direction. The experiments show that these processes, when the positron and the electron are also destroyed (annihilation), could happen with large probability if the electron and the positron have approximately the same momentum. Because the electrons in the matter have small momentum this means that the positron has to lose practically all of its energy before the meeting with the electron. Because the law of energy and momentum conservation is also true in annihilation this means that the gamma quanta leave the preparation move in the opposite direction and each photon has the energy that corresponds to the rest mass of one electron. Isotopes can decay by producing a positron besides the decay scheme, thus the two photons with 511 keV energy are also need to be taken into account.

3 Statistical feature of the decays

According to the Fig.1 in a $^{137}\text{Cs}$ sample with 1 kBq activity in 944 cases an excited $^{137}\text{Ba}$ nucleus with 662 keV energy and in 56 cases a ground state $^{137}\text{Ba}$ is created per second. From the 944 excited Ba nuclei 851 emit a photon with 662 keV energy and decays to the ground state while in 93 cases the excited nucleus interacts with one of the orbital electron of the atom (on K or L shell) and decay to the ground state by emitting a photon. This process is the inner conversion. The emitted photon is mono energetic, its kinetic energy is equal with the electron binding energy substrates from the nucleus nivel energy. In our case the emitted photon comes most likely from the K-shell which binding energy is approximately 32 keV.

Can we be sure that the number of emitted photons are EXACTLY 851? (The change of activity can be neglected because of the long – 30 years – half-life.) Because of the statistical feature of the decays only the expectation value can be declared, the actual number fluctuates around this. It is foreseeable that the fluctuations can be described
with the Poisson distribution. In case of Poisson distribution the standard deviation is the square root of the mean, in our case it is $\sqrt{851} = 29$ so the number of emitted gamma particles are between $852 - 29 = 822$ and $852 + 29 = 880$ with 67% probability. The relative uncertainty is $(880 - 822)/851 = 6.9\%$.

Let’s count the gamma photons which are emitted from the source in 100 s! It is expected that 85100 photons are going to be emitted so we can say that the number of the emitted photons will be between $85100 - \sqrt{85100}$ and $85100 + \sqrt{85100}$ with 67% probability. In this case the relative uncertainty is $(85392 - 84808)/85100 = 0.69\%$ so the tenth of the previous case. From the thought it is obvious if we want to reduce the relative uncertainty to $n$-th time we have to measure to $n^2$ times longer. As a summary, it can be noted that the activity gives only the expectation value of the number of the decays per unit time in the sample. Because of the statistical feature of the decays the actual number of the decayed nuclei can be determined only within limits based on the Poisson distribution. The larger the expectation of number of the decays per unit time, the smaller the relative uncertainty is.

4 Absolute activity measurement

We have stated above that we can investigate the kind, the expectation value and the energy of the emitted particle per unit time if we know the activity and the decay scheme. The object of the activity measurement is reverse: if we know the decay scheme and we can measure the number of specific kind of emitted particles with determined energy the activity can be measured. But this measurement raise some problems which have to be consider carefully.

4.1 Solid angle of the detection and the efficiency

The particles escape in random orientation in all directions from the decaying source. If we would like to observe all the particles that emitted from the sample we should have to surround it with detectors. This is possible if we can place the sample in the middle of the detection volume, to the inside of the detector. Detectors that cover the whole solid angle can be proportional chambers or liquid scintillator counters. These detectors are used partly in the professional activity measuring and authentication instruments. These are referred as $4\pi$ counter in the literature as a notation these detectors cover the whole solid angle.

In our measurement we use a sodium-iodide [NaI(Tl)] scintillator detector, which is placed completely outside of the sample. Therefore, just a part of the emitted gamma photons reach the detector. So we have to care with the effect of the smaller solid an-
gle. A detector with surface $F$ in a distance $R$ from the point-like sample is reached by $F/(4\pi R^2)$ part of the emitted photons. So the solid angle factor is

$$g = \frac{F}{4\pi R^2}. \quad (1)$$

We have to consider too the properties of the radioactive decay which are mentioned before. In case of the most of the isotopes not all decay leads to an emission of a determined kind of particle. For example we saw before that during the decay of $^{137}\text{Cs}$ out 1000 events only 851 is an emitted gamma photon with 662 keV energy. This intensity rate can be taken into account with introducing a factor $\epsilon$. $\epsilon < 1$ is usually true. But in the case of the positron decay this rate of the photons with 511 keV energy can be more than 1 because during a positron decay usually two photons are created.

Another problem is that the gammas, which reach the detectors, do not always give a signal in the detector in moreover not all the photons can be counted that give a signal. There are theoretical and technical issues: we know that in the interaction of the gamma photons is ruled by three processes: the Compton-effect, the photo electrical effect and the pair production. The experiments show all of these three process have a small probability so the gamma radiation has good penetrating ability. In other words: the most of the photons that reach the detector, do fly through the detector without any interaction so we cannot detect them. So the detection must have efficiency lower than 1 ($\eta < 1$). We have to be more precise with our consideration about the efficiency of the detectors. Because a single photon with energy $E$ can interact with the matter we get different detection efficiency if we ask how many photons that reach the detectors create Compton-effect, photoelectric effect or pair production. In all of the three process in the detector a new particle starts with unit charge, large energy and mass of an electron. Partly the particle stimulate the atoms and molecules on its way to emit a visible photon. Due to this, thousands or tens of thousands of visible photons could be created in our NaI(Tl) crystal. The number of the created photons are proportional to the energy that is left in the crystal. Considering that we collect the created photons for a few seconds the total energy can be left in the crystal not only with photo electrical effect but with multiple scattering.

In our measuring set-up the signals which come from the detector with different amplitudes go to the differential discriminator (DD). This instrument allows a signal to reach the counter with determined amplitude. Therefore, it depends on the actual settings of the DD, which signals (those that are caused by some process) are measured and which are not. In our measurement we count the events when the total energy of the photons remain in the detectors. This reduces the efficiency of the detection depending on the settings of the instrument. The unified efficiency of the detection ($\epsilon$) contains the solid angle factor ($g$), the intensity factor ($\epsilon$) and the detection efficiency ($\eta$) which is
determined by the actual settings of the instrument:

\[ e = g \eta. \]

The above mentioned practically means, that we have to measure this unified efficiency in any actual case. If a source emits only one photon with determined energy per decay and the activity of the source is \( A \), then the counter detect

\[ N = e \cdot A \cdot t \]

events during \( t \) time. Here \( e \) is the unified efficiency, \( N \) is the number of the signals, which are counted by the counters during \( t \) time. The counters are connected to the output of the DD. The optimization of the settings of the DD can be done based on the actual spectrum of the source. We note that the recording of the nuclei radiation are detailed in another labor practice (Gamma spectroscopy).

### 4.2 Activity measurement with coincidence method: elimination of the efficient factor

In the Eq.(3) \( N \) and \( t \) are measured so the \( e \cdot A \) can be determined. In our equation we have two unknown quantity: the unified counting efficiency \( e \) and the activity \( A \) which has to be determined. We search a method which allows to determined these. One of the possibilities is the coincidence method. This method can be used to measure sample which emits at least two particle in a short (technically in the same) time. We will measure the activity of a \(^{60}\text{Co} \) sample with this method. Here two photons are created in a gamma cascade process; the two photons follow each other in \( \approx 10^{-12} \) s.

Coincidence here means the coincidence of two or more signals in time. The time coincidence of the signals (usually electrical impulses) is measured by a coincidence instrument. It has two or more inputs and a single output. There is an impulse on the output only if signals arrive to the inputs in the same time. Of course, we have to declare how accurately we want the time coincidence of the signals. The \( T \) time interval within all signals are considered as same signal is the time resolution of the instrument. Obviously, the smaller is the time resolution the more precise the detection of the actual signal coincidence.

Reducing of the time resolution has technical difficulties. The instruments, which have more precise time resolution than \( 0.5 \) \( \mu \)s are called fast coincidence, the less precise instruments are called slow coincidence. In our measurements we use approximately \( 1 \) \( \mu \)s time resolution. The simplest coincidence circuit is a logical AND gate. If on the inputs of the circuit there are \( T_e \) long impulses the resolution will be \( T_e + T_e = 2T_e \). (Try to deduce this!)
### 4.2.1 The principle of the activity measurement of the $^{60}$Co

Let observe the two gamma photons that come from the $^{60}$Co isotope! The source emits $(2 \cdot A \cdot t)$ number of photons under $t$ time practically in the same time. We will see during the measurement that it is expedient to set the differential discriminators so that each detectors can detect both of the photons. The detector, which is marked as 1. or 2. is the unified efficiency of the detection of the photon which are marked as 1. or 2. is $e_{11}$ and $e_{12}$ or $e_{21}$ and $e_{22}$. So the number of the detected photons by the detector 1. or detector 2. during $t$ time is

$$N_1 = (e_{11} + e_{12}) \cdot A \cdot t$$

(4)

and

$$N_2 = (e_{21} + e_{22}) \cdot A \cdot t$$

(5)

These are called "collateral counts". How much will be the counts in coincidence with the two detectors? If both of the detectors can detect both part of the cascade than we will get coincidence in two case: if the 1. detector detects the 1. photon and the 2. detector detects the 2. photon or if the 1. detector detects the 2. photon and the 2. detector detects the 1. photon. So the number of the coincidence is

$$N_c = (e_{11} \cdot e_{21} + e_{12} \cdot e_{22}) \cdot A \cdot t$$

(6)

It is true if the detections are independent because in this case the probability of the coincidence is the product of the probabilities. We do not care which gamma photon is detected by one of our detector because real coincidence only could be if the other detector detect the other photon. The product of the Eq.(4) and Eq.(5) divided by the Eq.(6) – after some sort – can be written up as

$$\frac{N_1 \cdot N_2}{N_c} = A \cdot t \cdot \left( 1 + \frac{e_{11}e_{21} + e_{12}e_{22}}{e_{11}e_{22} + e_{12}e_{21}} \right).$$

(7)

The energy of the photons which are emitted by the $^{60}$Co isotope (more precisely by the daughter element of it, by the $^{60}$Ni) are 1.173 MeV and 1.333 MeV. These energies are quite near to each other so

$$e_{11} \approx e_{12} \text{ and } e_{21} \approx e_{22}. $$

(8)

As it has been noted before, if both of the total energy peaks are captured by the DDs the factor next to the product $A \cdot t$ is approximately 2. Then the desired formula for the activity is simply

$$A = \frac{N_1N_2}{2tN_c}. $$

(9)
But if we use this formula it has to be assumed there is no direction correlation between the two photons. It means that compared to the direction of one of the photon the other photon is emitted to all direction with the same probability. Consider that to prove our statement! In the case of the $^{22}\text{Na}$ we capture the photons with 511 keV energy we measure a lot of coincidence at 180° and less at 90° while the $N_1$ and the $N_2$ do not change (if the source is in the center). Therefore the activity which is calculated according to the formula depends on the angle. This is obviously a false result.

In consecutive gamma decays the emitted photons usually have some correlation therefore the number of the coincidence depends on the position of the detectors. The measured $^{60}\text{Co}$ sample’s direction dependence is small, there is no strong dependence on the angle. But we get different activity if we set our detectors at 180° or at 90°. This angle dependence depends on the spin and parity of the decaying nucleus and we get important physical informations about it by measuring the angle dependence with the coincidence method.

Of course the previous considerations can be applied to other decays too. For example to the case when a beta decay is followed by a gamma decay and one of our detector detects the electron and the other detects the gamma particle. In this case our derivation leads to a little different expression than the Eq.(9).

### 4.2.2 Random coincidences

We have mentioned that the coincidence instrument accepts the signals the same if they reach the detectors within $T$ time interval. If we use a radioactive source that emits only one photon in a physical process it can happens that two nuclei decay independently within $T$ time so these events may are detected by our instruments as a coincidence. Because this isn’t due to a physical cause it is called random coincidence.

Let determine the number of the random coincidences! Consider that we measure for $t$ time and during this one of our detector detects $N_1$ and the other detects $N_2$ collateral counts. In case of each impulse the coincidence instrument "observe" for $T$ time that if there is a signal from the other arm during this $T$ time. The probability of a random signal from the other arm is $T \cdot N_1/t$. The number of the random coincidence from $N_2$ counts is $N_2$ times this during $t$ time:

$$N_{\text{random}} = \frac{N_1 N_2 T}{t}. \quad (10)$$

Remember that in the case of circuit that we use $T = 2T_e$, where $T_e$ is the signal on the inputs of the coincidence instrument. Of course there are random events in addition to
the real. The measured number of the real coincidence is

\[ N_m = N_{\text{real}} - N_{\text{random}} \]  

(11)

To determine the real coincidence we have to subtract the number of the random coincidences \((N_{\text{random}})\) from the measured \((N_m)\).

We can use several methods to determine the number of the random coincidences:

a) \(N_{\text{random}}\) can be calculated from (10) by measuring the collateral counts if we know \(T\) resolution time which can be determined by the long of the signals on the input of the coincidence instrument. (This can be measured with an oscilloscope.)

b) We use a radioactive preparation where there is only one emitted photon for each decay (i.e. \(^{137}\text{Cs}\)). In this case there is only random coincidences so from the (10) the \(N_{\text{random}}\) and the collateral counts can be measured so \(T\) can be determined. If we know \(T\) we can calculate the number of random coincidences in the case when the sample has real coincidences too.

c) It is not necessary to measure with another isotope. If we electronically ruin the real coincidences then with the same settings we can measure the random coincidences directly. We can ruin the coincidence in a simple way: front of the coincidence instrument we apply a proper time delay. The real coincidence now cannot give a signal so all observed signals must come from random coincidence with signals of other processes. The advantage of this method is that all the settings of the measurement remain the same so the systematical errors are the least.

In our practice we use the c) method.

4.3 Measuring of the relative activity

In case of the relative activity we compare the activity of our sample to a benchmark isotope with known activity. As a condition of the measurement we have to use the same type of benchmark isotope then our sample with the same geometry (if it is possible). We measure the counts from the benchmark and from our sample. If all the settings are the same then the ratio of the counts is equal to the ratio of the activity. From this measurement the unknown activity can calculate easily.
4.4 Other factors that affect the measurements

4.4.1 The background

There is always a continuous natural radioactive background in our environment, which can be detected by our devices. If we ascribe to the sample this activity we overestimate the activity of our isotope. Therefore always measure the background activity with the settings which we will use. We have to be careful, the isotopes have to be far away from the detectors while measuring the background radioactivity. Hereafter the background always have to be subtracted from the measured number of counts of the sample. The background can change the result significantly in case of low activity sample. In our case there is a very little effect.

4.4.2 Dead time of the detectors

It is obvious while our system is busy with recording a signal it is not able to record another one. This time interval is called dead time. If a signal reach the detector during the dead time it cannot be recorded by our instrument. Therefore if we measure for \( t \) time actually the instrument measure for shorter time. In case of \( t \) dead time and \( N \) counted impulse the real measuring time is shorter than the \( t \) measuring time with \( t \cdot N \) so the real measuring time is

\[
  t_m = t - N \tau
\]  

The correction of the dead time \( t \) has to be substituted with \( t_m \) to all of the formulas before. We take note of that for the precise measurement even in the case of the relative activity we have to take care about the effect of the dead time correction because the activity of the benchmark and the sample are different the number of counts will be different so the correction will be different too.

5 The instrument

A draw about the instrument can be seen on Fig.2. In our measurements we use scintillator detectors that has NaI(Tl) crystal on a PMT. We won’t move one of the detector during the measurement (fixed detector), the position of the other detector can be varied between 20° and 180° (moving detector). The voltage of the detectors come from the same supply and it is approximately 820 V. The two arms has nearly the same amplification.

The amplifier and DDs of the two energy analizator arms are in a similar blocks. We can choose the desired total energy peak with the DDs. In differential mode the DD gives a signal if the signal is in it input range \( (V, V + dV) \). The \( V \) is called the basic level and it
Figure 2: The block scheme of the instrument. (S – the gamma source, NaI – the scintillator, PMT – photo multiplier, FC – fitting circuit, A – amplifier, DD – differential discriminator, SF – time-delay signal former device, Co – coincidence device, C – counter)

can be set with a potentiometer gently between 0.1 V and 10 V. The $dV$ is the range of the channel. Its value can be modified with a similar potentiometer between 0.01 V and 1 V so in case of this ten-times-turnable potentiometer here is a tenth times deviation compared to the previous one. We can measure the amplified signals with an oscilloscope and the uniform impulse that comes from the output of the DDs. We divide to two parts the signals that come from the DDs and on one of the arm we count the collateral counts ($N_1$ and $N_2$). On the other arm the signal of the fixed detector directly, the signals of the moving detector after some delay go to the coincidence instrument and we count the output signals of the coincidence instrument ($N_c$ and with delay $N_{\text{random}}$ – see Eq. (9), (10), (11)). The measuring time can be set with the fourth counter of the instrument.

After setting we can start the measurements with one touch and the instrument measure the time, $N_1$, $N_2$, $N_c$ or $N_{\text{random}}$ simultaneously depending on the presence of a potential time delay. The measurement is supported by an oscilloscope with two channels. With this the form, the size and temporal properties of the signal can be investigated.
Figure 3: The spectrum of the $^{22}$Na sample. The data come from a labor practice. The first peak belongs to the 511 keV energy. The x-axis can be calibrated with this well-known peak. The second smaller peak belongs to the 1280 keV energy. (See Fig. 1)

5.0.1 Notation about the measurement

We use low activity source during the practice. If we stand in half meter distance from the sample we will get the vanishingly small part of the exposure limits during the practice time – four hours. Still we must not brake the radiation protection rules! Do not touch the source with bare hands but only with a clip or forceps. The reduction of the distance raise the risk to get major amount of radiation in our hand.

6 Measurement tasks

6.1 Measurement tasks for the absolute activity

In the labor practice there are the following tasks:

1. Measure the spectrum of the $^{22}$Na with the one-channel differential discriminator. An example plot can be seen in Fig. 3.

   • Set the range of the channel of the DDs to 0.1 V!
   • Set the measurement time to 0.2 min (12 s)!
   • Change the basic level of the DDs by 0.1 V step-by-step to measure the spectrum of the $^{22}$Na in both of the detectors. ($N_1$ and $N_2$, we do not need the
Figure 4: The angle dependence of the counts in case of the $^{22}$Na. The data come from a labor practice. Because of the annihilation process the number of the counts depends on the position of the detectors very strongly. We use the $^{22}$Na sample to calibrate the x-axis but we cannot measure its activity. (But we calculate the activity at 90° and at 180° anyway to demonstrate that the coincidence method cannot be used in this case.)

Figure 5: The spectrum of the $^{60}$Co sample. The data come from a labor practice. There are two peaks that belong to the 1173 keV and to the 1333 keV energy. (See Fig.1) The x-axis can be calibrated with the spectrum of the $^{22}$Na.
number of the coincidence in this measurement.) Attention: with the same potentiometer value the range of the channel is only the tenth!

2. After the recording the spectrum set the basic level and the range to capture the 511 keV energy annihilation gamma peak (Fig.3). With the 0.2 min (12 s) measuring time vary the angle of the moving detector and obtain the angle dependence of the number of the coincidences and the collateral counts. Plot the number of the coincidences as a function of the angle. Start at 135° and move to 225° with 10° steps and one more additional value at 180°! An example measurement can be seen in Fig.4.

3. Leave the moving detector in the 90° position and set the measurement time to 10 min. Measure the collateral counts and the number of the coincidences without and with 10 ms time delay \( (N_1, N_2), \text{without/with time delay } N_c/N_{\text{random}} \). Explain the results! How it is possible to get different result with and without time delay?

4. Set the measurement time to 0.2 min again and measure the spectrum of the \( ^{60}\text{Co} \) in both of the detectors. See Fig. 5.

5. Set the basic level and the range according to the measured \( ^{60}\text{Co} \) spectrum as it mentioned before in this note. Set the measuring time to 15 min! Measure twice in the 90° position and twice in the 180° position with and without time delay. Obtain the activity in the two different position with the corresponding error, and yield the average and the standard deviation of the measured activity.

6. From the average activity obtain the expose which we get during the labor practice if we stand from the source at 1 meter! The dose constant for the \( ^{60}\text{Co} \) is 305.

7. Calculate when the \( ^{60}\text{Co} \) had 1 MBq activity? The \( ^{22}\text{Na} \) sample had 1 MBq activity at the same time. Which activity have the \( ^{22}\text{Na} \) does now?

8. According to the previous calculation obtain the resolution of the instrument!

9. Measure the (probably different) dead time of the detectors! Set the basic level of the discriminator to 0.2 V and change the mode to integrating mode (INTG), set the measuring time to 5 minutes at least! We have to
   - measure the counts per unit time in case of the first source \( (N_1) \)
   - measure the counts per unit time in case of the first and the second source \( (N_{12}) \)
   - measure the counts per unit time in case of the second source \( (N_2) \)
   - measure the background \( (N_{\text{background}}) \)
From this four measurements (perhaps from the first three) obtain the dead time of the detectors. We note that because of the dead time $N_{12} < N_1 + N_2$. If the value of the $N_1 + N_2 - N_{12}$ is approximately the value of the $N_{\text{background}}$ then there is a significant difference between the dead times. This depends on whether we calculate with the background or not. Notations:

- $n_1$ The number of the particles that come from the first source and reach the detector per unit time
- $n_2$ The number of the particles that come from the second source and reach the detector per unit time
- $h$ The number of the particles that come from the background and reach the detector per unit time
- $\tau$ The dead time of the detector per unit time

Then the following equations can be set-up to the measured counts:

$$N_1 = (n_1 + h)(1 - N_1 \tau) \quad (13)$$
$$N_2 = (n_2 + h)(1 - N_2 \tau) \quad (14)$$
$$N_{12} = (n_1 + n_2 + h)(1 - N_{12} \tau) \quad (15)$$
$$N_{\text{background}} = h(1 - N_{\text{background}} \tau) \quad (16)$$

From these equations if $N_1 + N_2 - N_{12} << N_{\text{background}}$ a second order polinom can be derived to the dead time $\tau$ and we have to find its roots. If $N_1 + N_2 - N_{12} \approx N_{\text{background}}$ then we have to find the roots of a third order polinom.

### 6.2 Control questions

1. Determine that how many photons and with how much energy will be emitted by the sources in the Fig.1 if each of the samples have 5 kBq activity!

2. Let’s carry out a test measurement! We detect 100 counts from the background during 10 s. If there is the sample we detect 120 counts. How long we have to measure if we want to determine the number of the emitted particles with 1% precision? Give the error of the estimated time too!

3. What kind of detector we use to detect the gamma photons?

4. What is the role of the differential discriminator (DD)?

5. On what effects does the unified efficiency depend?

6. How we can eliminate the unified efficiency from our results?
7. Which (mentioned) isotope(s) cannot be measured with the coincidence method? Why?

8. How we can thread the random coincidence?

9. If we count in the first detector $N_1 = 1000$ and in the second detector $N_2 = 2000$ counts during $t = 500$ s while we know that there is $N_{\text{random}} = 10$ then how much the activity is?

10. Why there is angle dependence in the case of the $^{22}\text{Na}$?

11. How can be the intensity rate factor ($\epsilon$) larger then 1?

12. What are the three processes that can happen in the matter of the detector?

13. What does inner conversion mean?

14. The $^{60}\text{Co}$ sample doesn’t emit the two photon in the same time. Why we can use the coincidence method to measure its activity?

15. If on the inputs of the coincidence unit there are $T$ long impulses what will be the resolution?

16. Why we can assume that the unified efficiencies are approximately equal ($\epsilon_{11} \approx \epsilon_{12}$ and $\epsilon_{21} \approx \epsilon_{22}$)

17. Which sources can cause random coincidences?

18. What does the dead-time of the detector mean?

19. What is and how we use the NaI(Tl) scintillator?

20. Which is the simplest coincidence circuit?

References

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