The measurement of the β decay

1. Introduction – the β decay and the weak interaction

This laborpractice is about the investigation of the β decay of the isotopes ^{137}Cs and the ^{90}Sr. The aim of the practice is to determine of the maximal energy of decays which is possible to do with the Fermi-Kurie transformation described in sec. 3.2.1. It is possible to determine the so-called ft-value, if we know the maximal energy.

1.1. The short history of the β decay

In the beginning of the 20th century, three types of radioactive decays were known: α, β and γ decay. In the α decay an α particle is emitted from the nucleus which is a bound state of two neutron and two proton: a helium nucleus. In the β decay an electron or a positron (the anti-particle of the electron) is emitted (at least that was known at this point) and in the γ decay a γ particle is emitted which is a photon. All of these were based on the observations of the experimental physicists. At this time, there were two atomic model, two idea how the atoms are built: the Rutherford model and the Thomson model. The Rutherford experiment proved that the Thomson model is wrong.

The Rutherford experiment brought up the question then: where the electron comes from in the β decay? When the Rutherford experiment was performed, the field theory was not used by particle physicists so they did not know that the electron is created „on the spot".

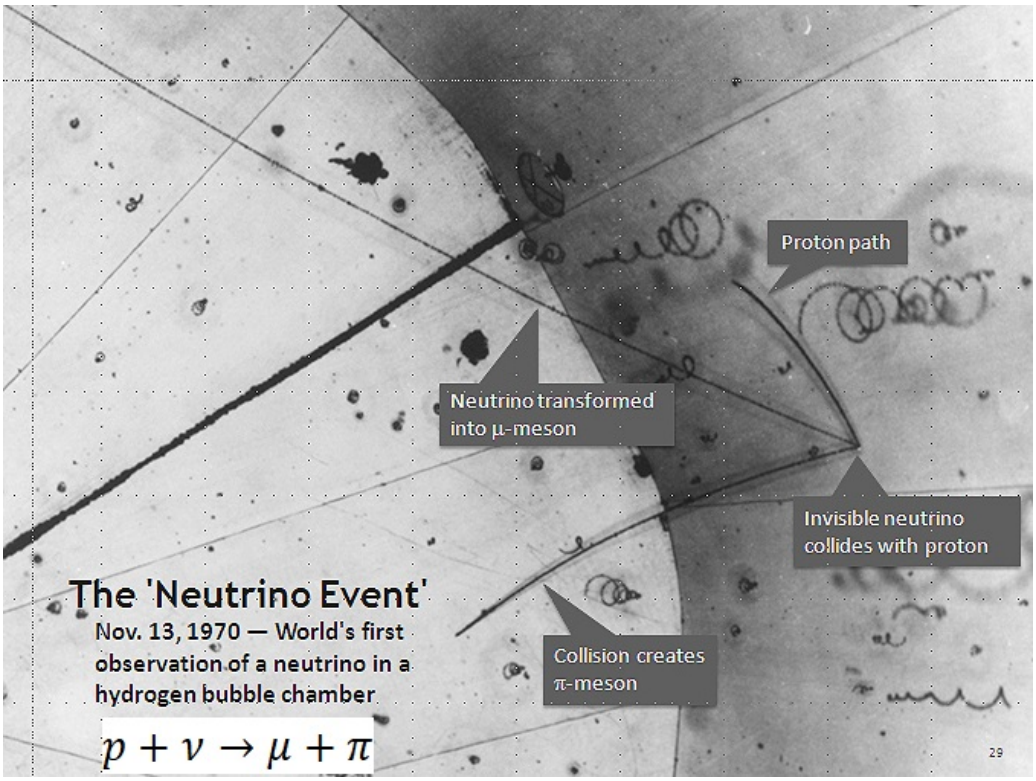
Another surpising observation was done by Hahn, Schmidt and Meitner in 1907. They measured that the electron/positron emmitted in β decay has no determined energy; instead the spectra was continuus and did not contain any peak-like structure as in the γ decay spectra. An example spectra can be seen in Fig. 5. Why is this observation suprising? Because if just one particle is emitted then it should have the energy of the difference of the energy of the initial and final nucleus which is always well determined because of the conservation of the energy. Then, why there is no peaks in the β spectra?

1.2. A new interaction and the neutrino hypothesis

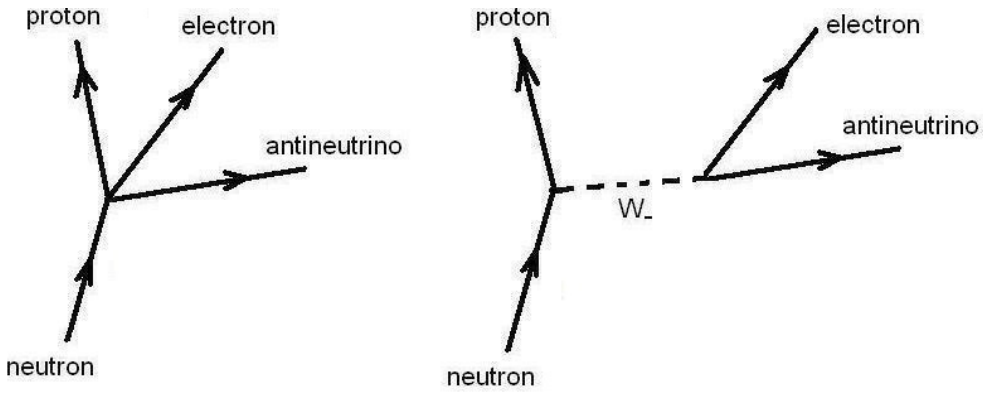
Among other explanations, there was Pauli's neutrino hypothesis. He assumed the existence of a chargeless and very light particle which could carry away energy. If we only measure the electrons then we will observe a continuous spectrum because the energy differenc of the initial and final nuclear states are distributed between two particles. The first observed neutrino was detected on 1970 as it is shown in Fig. 1.

The first theoretical model for the creation of the neutrino was proposed by Enrico Fermi in 1933. An essential graph of the process is shown in Fig. 2.

The more precise mathematical investigation proved that the Fermi theory should be improved. These studies introduced other new particles, the gauge bosons which have the same role in the weak interaction as the photon has in the electromagnetic interactions but these bosons are much heavier than the known particles that time. They are approximately 80 times heavier



1. ábra. The first observed neutrino.



2. ábra. The Fermi theory of the β decay.

than the proton and called $W^{\pm}, Z^0$ bosons. With the photon they are the four gauge bosons of the electro-weak interaction which unifies the electromagnetism with the weak interaction.

1.3. The types and the energetic aspects of the β decay, allowed and forbidden transitions

Three types of the β decay has been observed which can be characterized by their energetic properties. Let the Q means the energy difference between the initial and the final states:

$$Q = (M_i - M_f - M_{e^-} - M_{\overline{\nu}_e})c^2, \tag{1}$$

where M_i is the mass of the initial nucleus, M_f is the mass of the final nucleus, M_{e^-} is the mass of the electron and $M_{\overline{\nu}_e}$ is the mass of the neutrino which will be considered to be zero since the current experimental limit is approximately $M_{\overline{\nu}_e} < 1$ eV. The three types of the β decay:

- β^- decay: ${}^A_Z X \rightarrow {}^A_{Z+1} Y + e^- + \tilde{\nu_{e^-}}$
  so the Q is:

  $$Q = (M_i - M_f)c^2$$

- β^+ decay: ${}^A_Z X \rightarrow {}^A_{Z-1} Y + e^+ + \nu_{e^-}$
  so the Q is:

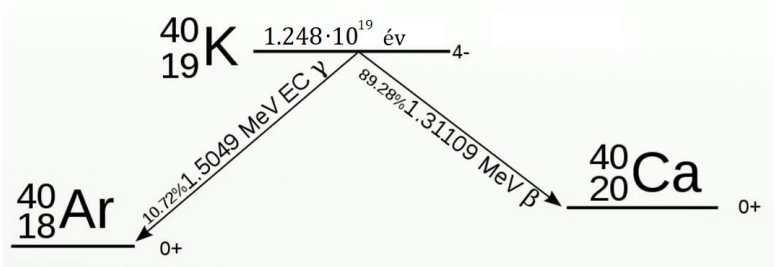
  $$Q = (M_i - M_f + 2M_{e^-})c^2.$$

  The extra term is because the atomic number of the final state nucleus is lower by one and the positron also have to created. This has a simple but important consequence: $Q > 1022$ keV mustbe true so there must be at least 1022 keV energy in the system to be able to decay with β^+ decay. An example is the β decay of the ${}^{40}K$ as seen in Fig. 3. The probability of the decay channel β^+ is lower than the probability of the β^- decay.

- e^- capture: ${}^A_Z X + e^- \rightarrow {}^A_{Z-1} Y + \nu_{e^-}$
  This process has the same energetic propertied than the β^- decay:

  $$Q = (M_i - M_f)c^2$$

  As a summary, we can observe that for the β^- and for the electron capture there is no minimum required energy but for the β^+ decay there is.



3. ábra. The simplified scheme of the β decays of the ${}^{40}K$.

The probability of a β decay are determined not only by the energetic properties but by quantummechanical selection rules. The selection rules are include the change of the spin $\Delta J = |J_i - J_f|$ and the change of the parity $\Pi = \pi_i \cdot \pi_f$ in the process. If the initial and the final parity is the same then $\Pi = 1$ otherwise it is $\Pi = -1$. As an example, let see one of the β transition of the ^{40}K. It decays from a 4^- state to a 0^+ state, so $\Delta J = 4$ and $\Pi = -1$. The transitions can be categorized as follow:

- $\Delta J = 0, 1$ és $\Pi = +1$: $0^+, 1^+$, allowed
- $\Delta J = 0, 1, ...$ és $\Pi = (-1)^{\Delta J}$: $0^-, 1^-, 2^+, 3^-, 4^+, ...$ ΔJ order, forbidden non-unique: FNU
- $\Delta J > 1$ és $\Pi = (-1)^{(\Delta J - 1)}$: $2^-, 3^+, 4^-, ...,$ $(\Delta J - 1)$ order, forbidden unique: FU

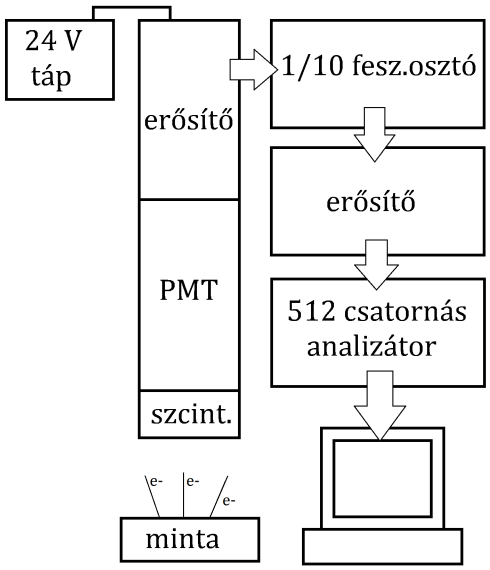
According to this categorization, the transition of the ^{40}K is 3^{rd} order FU transition.

The probability of the transitions are obviously related to the order of the forbiddeness. If the order is low then the probability of the process is higher; in other words the half-life of the isotope is less than in a higher order transition. In the laborpractice, the transition are forbidden in a certain order which is summarized, with other infos in Tab. 1..

2. The experimental setup

The schematic drawing of the experimental setup can be seen in Fig. 4. The role of the different parts are the following:

- Source: During the measurement, we investigate the ^{137}Cs és ^{90}Sr isotopes. The cesium is for the calibration. The maximal energies of the strontium should be determined.
- Scintillator: The β scintillator crystal usually made of lower atomic number material which has good scintillation properties. They are made to be quite thin because the shortness of the mean free path of the electron in the material.
- Photo mutlipier tube: The PMT multiplies the photons which are created in the scintillation crystal.
- Erősítő: Amplifies the signal coming from the PMT.
- 1/10-es voltage divider and amplifier: Prepare the signal to be processed by the analizator.
- Multichannel analizator: MA divides the signals into 2^n numbers of channels by their energy.



4. ábra. The block diagram of the experimental setup.

3. The energy spectrum

In the 2. we saw the experimental setup which is able to measure the electrons coming the β decay of the ^{137}Cs and the ^{90}Sr isotopes. In this section we will go trought the form of the energy spectra ignoring the derivation of the formulas which can be found elsewhere.

3.1. The origin of the energy spectrum

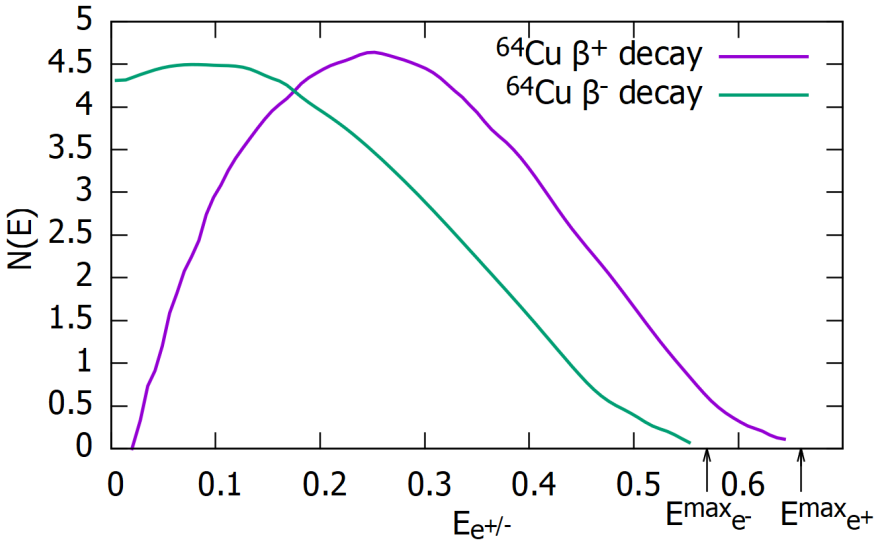
The energy spectrum of the emitted electron can be derived from Fermi's Golden rule. The partial decay constant can be written as:

$$\lambda(E_e) = \frac{2\pi}{\hbar}|V_{f,i}|^2\rho(E_f)|_{E_e}$$

where $V_{f,i}$ is the ineraction matrix element and $\rho(E_f)$ is the final state energy density at a given electron energy. These two quantity should be calculated in details but we just give the results.

3.2. A mag–β-részecske Coulomb kölcsönhatása

There is a final state effect which should be considered: the Coulomb-interaction of the electron/positron and the final state nucleus. There is a difference in the spectra of the electrons and positrons since the nucleus is positively charged so it repels or attract the positron or the electron. This difference can be seen in Fig. 5. This final state Coulomb interaction is described



5. ábra. The energy spectra of the ^{64}Cu negative and pozitive β decay. (R.D.Evans, The atomic nucleus (New York: McGraw-Hill, 1955))

by the Fermi function[1].

$$F(Z, E_{\text{kin}}) = \frac{2(1+S)}{\Gamma(1+2S)^2}(2\rho p)^{2(S-1)}e^{\pi\eta}|\Gamma(S+i\eta)|^2 \tag{2}$$

where $S = \sqrt{1-\alpha^2Z^2}$ (α is the fine-structure constant), $\eta = \pm\frac{\alpha ZE}{pc}$, a + sign for the electron and – sign for the positron, a $\Gamma(\cdot)$ is the Gamma function, a $\rho = \frac{r_N}{\hbar}$ is the effective diameter (r_N the diameter of the final state nucleus) which is characterized by a parametric form[2]:

$$\rho = 0.0029A^{1/3} + 0.0063A^{-1/3} - 0.0017A^{-1}. \tag{3}$$

[1] https://en.wikipedia.org/wiki/Beta_decay#Fermi_function

[2] L.R.B.Elton. A semi-empirical formula for the nuclear radius. NuclearPhysics,5(0):173 178,1958

This Coulomb-correction is implemented in Python in Sec. 4.

With this the decay constant is:

$$\boxed{\lambda(E_e) = g^2 |M_{f,i}|^2 \frac{1}{2\pi^3 \hbar^7 c^6} F(Z,E) S_n(E,p) (E_{\max} - E_e)^2 \sqrt{E^2 - (mc^2)^2} E_e dE_e.} \tag{4}$$

Now, how to measure this?

3.2.1. The Fermi-Kurie transformation

The decay constant describe th probability of a β transition at E_e energy. So, if we unify all the constant terms in Eq. (4) to a K constant we can yield the energy distribution in the following form:

$$N(E) = K F(Z,E) S_n(E,p) (E_{\max} - E_e)^2 \sqrt{E_e^2 - (mc^2)^2} E_e.$$

We assume $S_n(E,p) \approx const.$ so $K' = K \cdot S(E,p)$. By reordering the terms

$$\boxed{\sqrt{\frac{N(E)}{K' F(Z,E) \sqrt{E_e^2 - (mc^2)^2} E_e}} = E_{\max} - E_e.} \tag{5}$$

On the left hand side of the equation there is the measured $N(E)$ distribution which become a linear function with negative slope of written in tha above form. The interception of the linear function with the x-axis is the maximal energy. This linear function is caled the Fermi-Kurie linear [3].

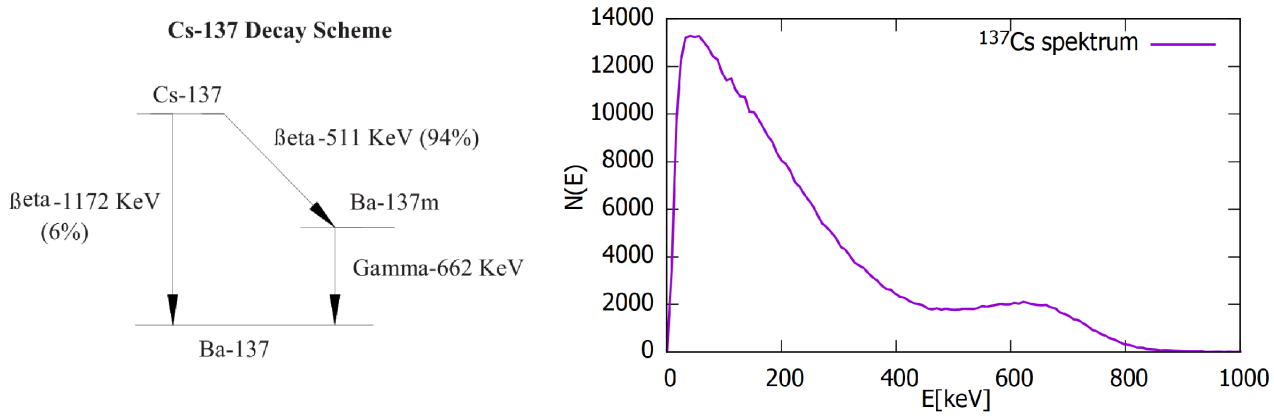
3.3. Exercises for the report

The calibration is done by the conversion peak of the ^{137}Cs isotope and the validation is done by the β^- decay in the spectrum with maximum energy 511 keV, see Fig. 6. (The larger, 1172 keV transition is neglected during this measurement.) On right hand side of Fig. 6., at lower energy the monotonically decreasing part is corresponds to the β decay. The plateau-like part is the peak of the conversion electron.

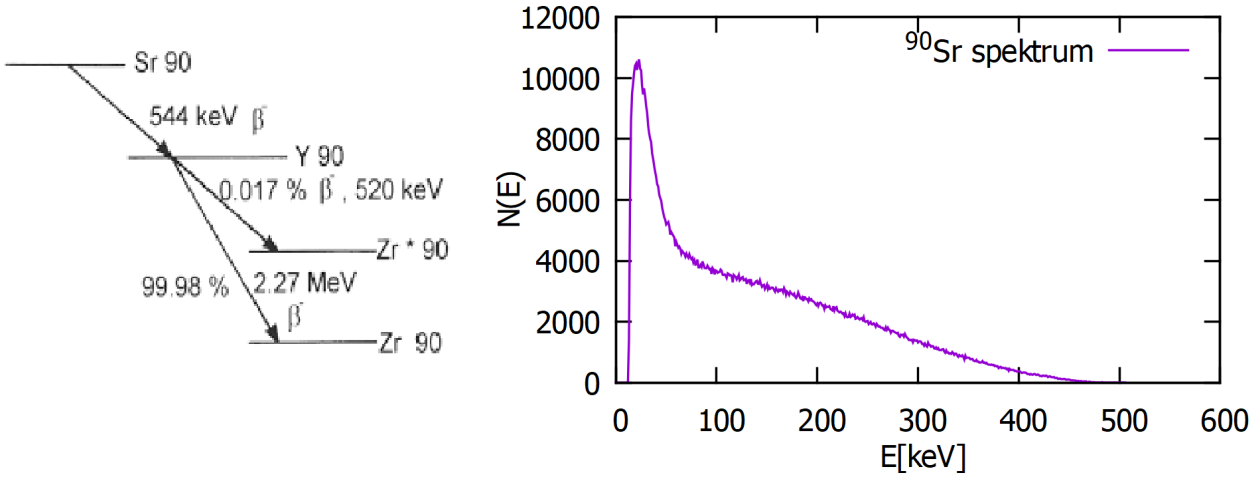
The calibration process: Because the position of the 630 keV conversion peak is uncertain, it should be validated. We guess the position then perform the Fermi-Kurie transformation and try to fit the linear part of the transformated spectrum. If the intercept, the maximum energy is around the nominal value we accept our guess for the peak position. If the intercept is not acceptable then we modify our gues of the position of the peak and do the process again. The iteration should be done until the intercept is close enough to the nominal value. As I mentioned before, the second point of the calibration is the first non-empty bin which corresponds to the 1 keV.

After that we can investigate the more complicate spectrum of the strontium.

[3] Kurie was not related to the Curie family: https://en.wikipedia.org/wiki/Franz_N._D._Kurie.



6. ábra. The decay scheme of the ^{137}Cs is on the left and the energy spectrum $N(E)$ is on the right.



7. ábra. On the left hand side, there is the decay scheme of the ^{90}Sr. On the right hand side, there is the $N(E)$ spektrum.

On the decay scheme, it can be observed, there are two β decay. In the spectrum it is also visible that the lower energy transition is between somewhere 300-1000 keV and the larger energy transition is around 1200-1700 keV. If we have to correct calibration then we can do the following: we perform the FK transformation defined in Eq. (5) and fit the decay with the larger maximal energy, because to that distribution the lower energy part has no contribution. After we determine the larger maximal energy, we can trasform back the spectra and the larger energy part can be substracted. After that we can transform the remaining spectrum and fit the lower energy part. In this manner, we can determine the both of the maximal energy.

The precision of the apparate is quite low, so 5-20% error is normal.

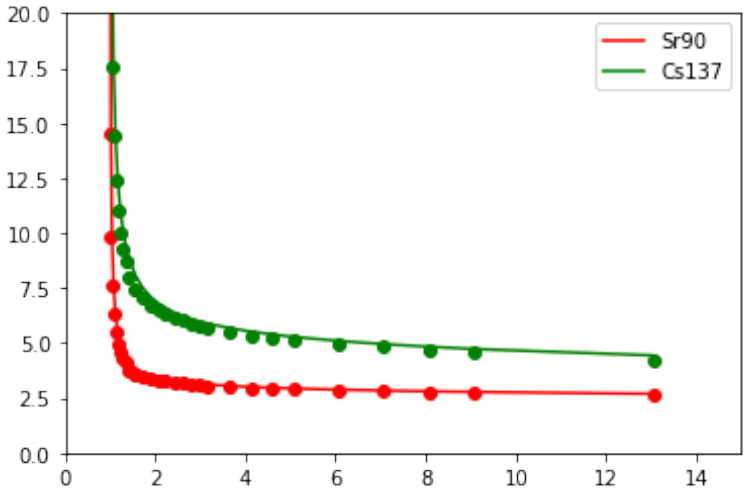
The report should contain the following:

- Perform the calibration and plot the original and the transformated spectra.

- Determine the maximal energy for the ^{137}Cs isotope.
- Determine the maximal energy for the ^{90}Sr isotope.
- Additional, optional task: determine the ft and the $\log(ft)$ vaules for the transitions!

4. Codes and tables

The Python code below implements the $F(Z, E)$ Coulomb-correction for the investigated cesium and strontium isotopes. The arguments for the function are `Z` atomic number, `A` atomic mass number and the sign `sign`, which $+1$ for electron (β^- decay) and -1 for positron (β^+decay). The `W` is the variable which is the electron energy in electron mass unit. The Coulomb correction and a few points from a numerical table can be seen in Fig. 8.



8. ábra. The Fermi Coulomb correction function for $^{137}_{55}$Cs and for $^{90}_{38}$Sr.

```
from pylab import *  ## Basic stuff
import cmath  ## Math function
import matplotlib.pyplot as plt  ## Plotting stuff
import numpy as np  ## Basic numerical packages
from scipy.special import gamma, factorial  ## For the gamma function

alpha = 1./137.036 ## fine structure constant
me = 9.1e-31 ## in kg ## electron mass in SI
c = 3e8 ## in m/s ## speed of light in SI
pi = 3.14159 ## pi
MeV = 1.6e-19*1e6 ## for conversion from SI to MeV
m = me * c * c / MeV ## electron mass in MeV

def FermiFunc(Z,A,sign,W):
    p = sqrt(W*W-1)  ## momentum in electron mass unit

    ## For rho: Elton's formula
    rho = 0.0029*pow(A,1./3.)+0.0063*pow(A,-1./3.)-0.0017/A

    S = sqrt(1-alpha*alpha*Z*Z)
    eta = sign*alpha*Z*W/p
    prefactor = 2.*(1.+S)/(gamma(1.+2.*S)*gamma(1.+2.*S))
    mod_sq_gamma = abs(gamma(S+complex(0,1)*eta)*gamma(S+complex(0,1)*eta))
    F = prefactor * pow((2.*rho*p),(2*(S-1))) * exp(pi*eta) * mod_sq_gamma

    return F
```

	^{57}Co	^{90}Sr	^{137}Cs
$M_{f,i}$	$1.04 \cdot 10^{-4}$	$1.47 \cdot 10^{-5}$	$6.72 \cdot 10^{-7}$
$\log_{10}(ft)$	7.7	9.4	12.079
t (half life)	271.74 day	28.79 year	30.08 year
Forbiddenness	1^+, allowed	2^-, FU	4^- FU

1. táblázat. Some information on the isotopes of the labor practice.