

HEAVY-ION COLLISIONS ON A QUANTUM COMPUTER

Experimental nuclear and heavy-ion physics seminar

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Mátyás Molnár, Eötvös Loránd University

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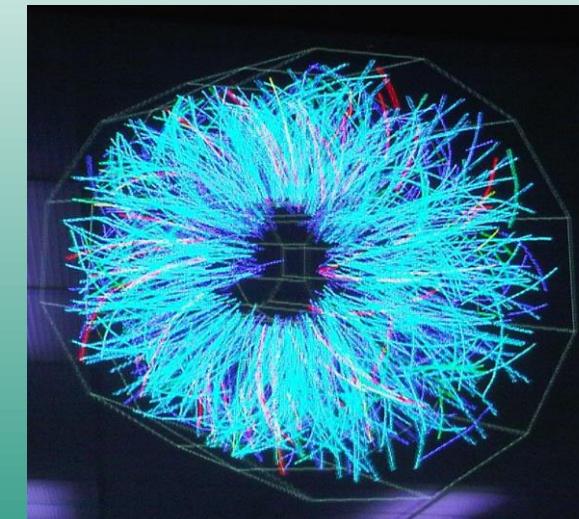
OUTLINE

1. Brief intro on quantum computing
2. VERY brief intro on open quantum systems
3. Quantum simulation of open quantum systems in heavy-ion collisions



IBM Q System One – 20 supercond. qubits

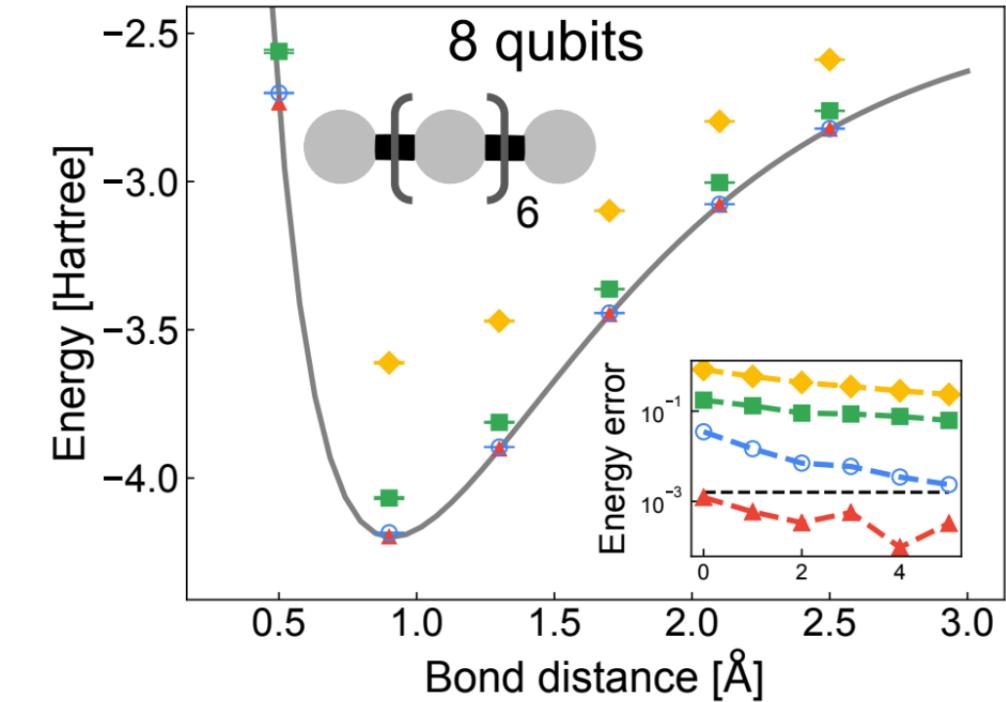
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Collision event at STAR

WHAT IS A QUANTUM COMPUTER GOOD FOR?

- Simulation of quantum systems!
 - Small molecules, quantum magnets... → variational quantum eigensolver [1]
- Some classical problems as well
- Some calculations exponentially faster
 - Break encryption schemes (e.g. RSA)



Google AI Quantum and Collaborators*, †, et al. "Hartree-Fock on a superconducting qubit quantum computer." *Science* 369.6507 (2020): 1084-1089.

QUBIT

- Classical bit: either 0 or 1

- Qubit: 0 and 1

$$|\Psi\rangle = a|0\rangle + b|1\rangle,$$

where $|a|^2 + |b|^2 = 1$

- After measurement:

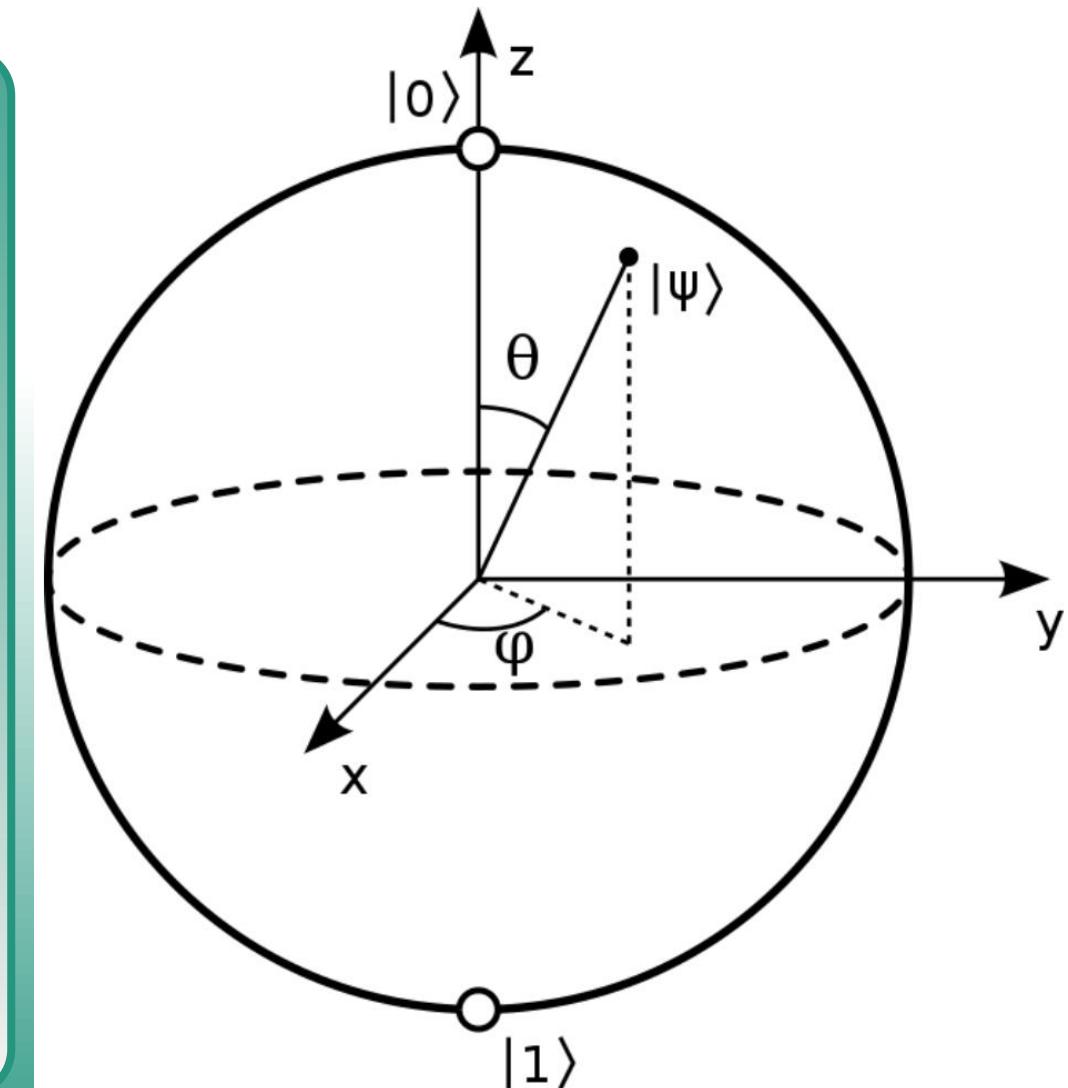
$$|\Psi\rangle = |0\rangle \text{ with probability } |a|^2$$

$$|\Psi\rangle = |1\rangle \text{ with probability } |b|^2$$

- Bloch sphere:

$$|\Psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right)|1\rangle$$

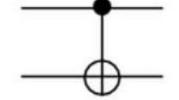
- Quantum computer efficient: N qubit Hilbert space has a dimension of 2^N

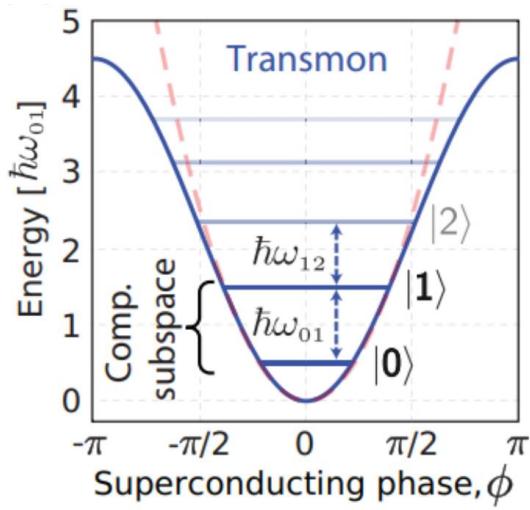


QUANTUM COMPUTER REQUIREMENTS

- DiVincenzo's criteria (just like Neumann's for classical!) [2]:
 1. A physical system with well defined, scalable qubits
 2. Preparation of an initial state
 3. Long coherence times
 4. Quantum gates
 5. State readout
...several physical implementations

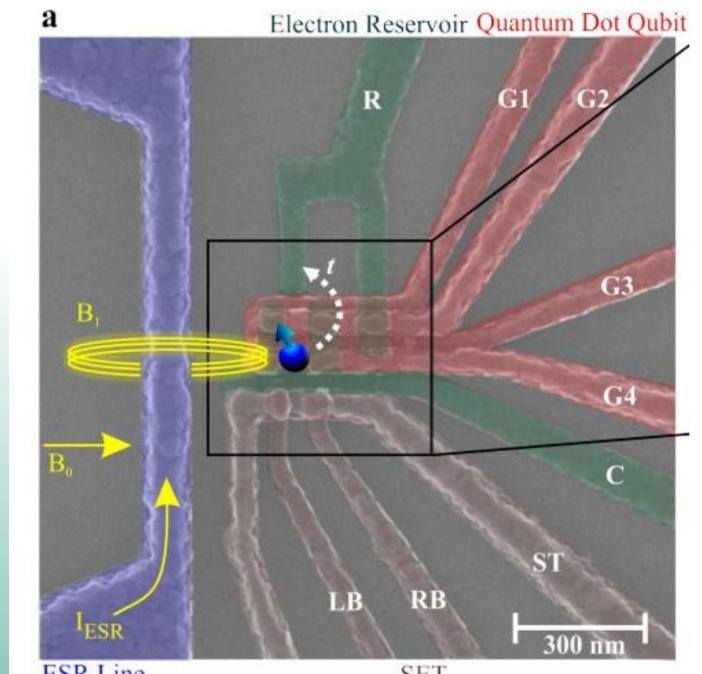
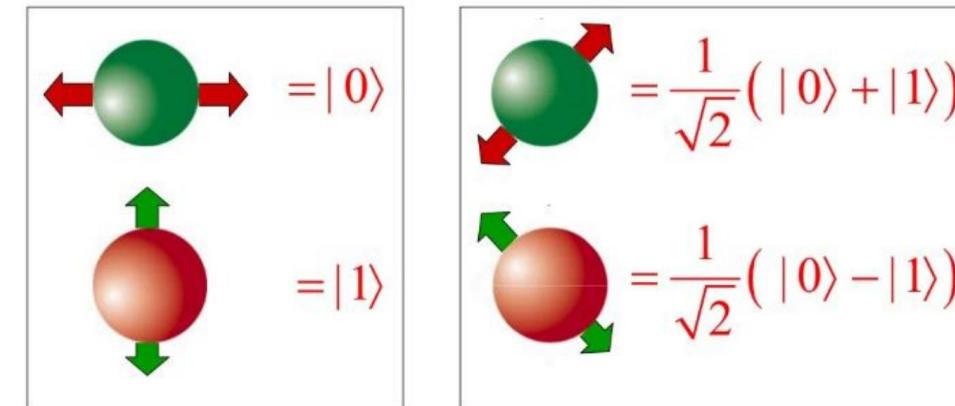
$$\rho(t) = \begin{pmatrix} 1 - \rho_{11}e^{-t/T_1} & \rho_{01}e^{-t/T_2} \\ \rho_{01}^*e^{-t/T_2} & \rho_{11}e^{-t/T_1} \end{pmatrix}$$

\boxed{X}	\boxed{Y}	\boxed{Z}	\boxed{H}	
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$



Krantz, Philip, et al. "A quantum engineer's guide to superconducting qubits." *Applied Physics Reviews* 6.2 (2019): 021318.

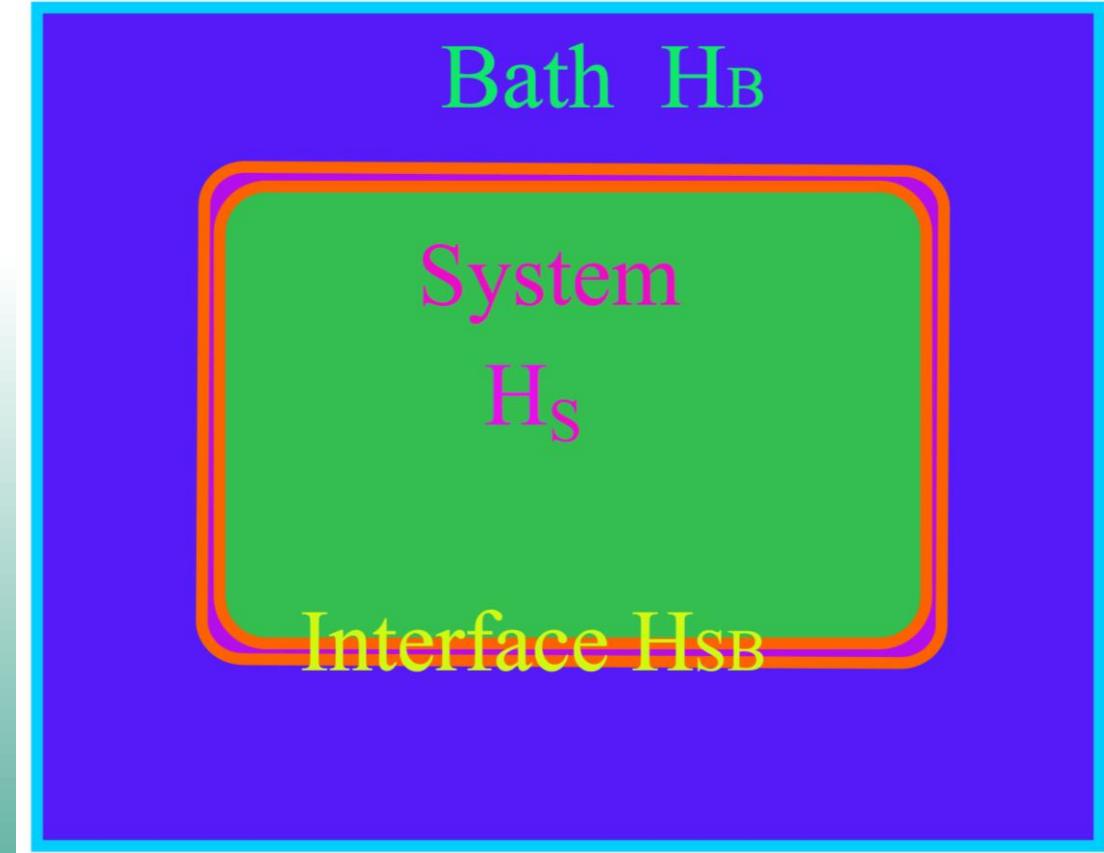
Photon polarization as a qubit



Veldhorst, M., et al. "An addressable quantum dot qubit with fault-tolerant control-fidelity." *Nature nanotechnology* 9.12 (2014): 981-985.

OPEN QUANTUM SYSTEMS

- QM system that interacts with the *environment/bath* (an external quantum system)
- Due to interactions: quantum dissipation [3]
 - Information contained in the system is lost to environment
 - Even if the combined system is in pure state $\rightarrow \Psi$
 - Subsystem cannot be described by a wavefunc.



OPEN QUANTUM SYSTEMS: OBSERVABLES, DYNAMICS

- Density operators ρ (by J. v. Neumann, L. Landau; see: manybody systems & stat. phys @ MSc 1st semester)
- Expectation value of observable A via scalar product [4]

$$(\rho \cdot A) = \text{Tr}(\rho A)$$
- Time evolution
 - closed quantum systems: unitary operators acting on the system
 - open: unitary op. not enough → effective equations of motion (*master equations*)
 - Phenomenological 1st-order differential equations, in form:

$$\frac{d\vec{P}}{dt} = \vec{A}\vec{P}$$

...very complicated, e.g. Lindblad eq.

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2} \{ L_i^\dagger L_i, \rho \} \right)$$

L_i jump operators: desc. dissipative dynamics

SOLVING THE MASTER EQUATIONS

- Observables:
 - (loss of) energy: *quantum dissipation*
 - loss of coherence: (robustness of) *quantum decoherence* [5]

1. System + environment as a closed system

- Time evolution: unitary trafo by global Hamiltonian

$$H = H_{System} + H_{Bath} + H_{System-bath\ interaction}$$

- State via partial trace

$$\rho_S(t) = \text{Tr}_B(\rho_{SB}(t))$$

2. Markovian system

- System has no memory of previous states (next state only depends on current)
 - Works if the system has enough time to relax to equilibrium before being perturbed again by interactions with its environment.
-
- Further to learn on the topic here: Open and non-equilibrium systems (L. Oroszlány), Open quantum systems (Máté Veszeli?)

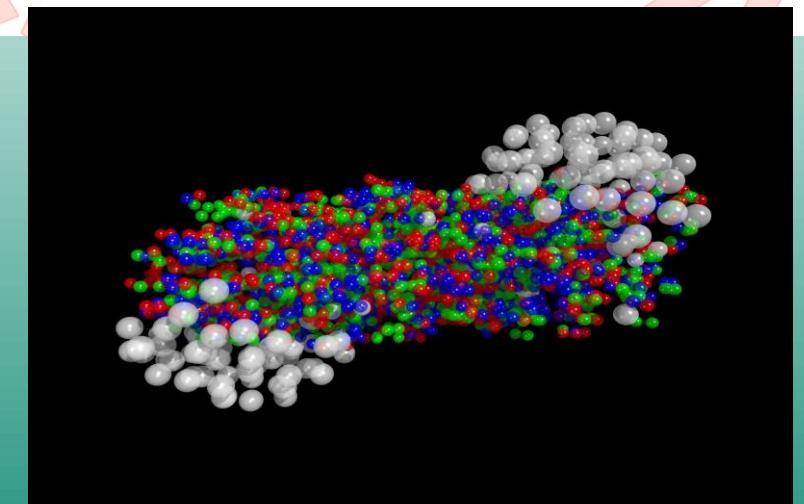
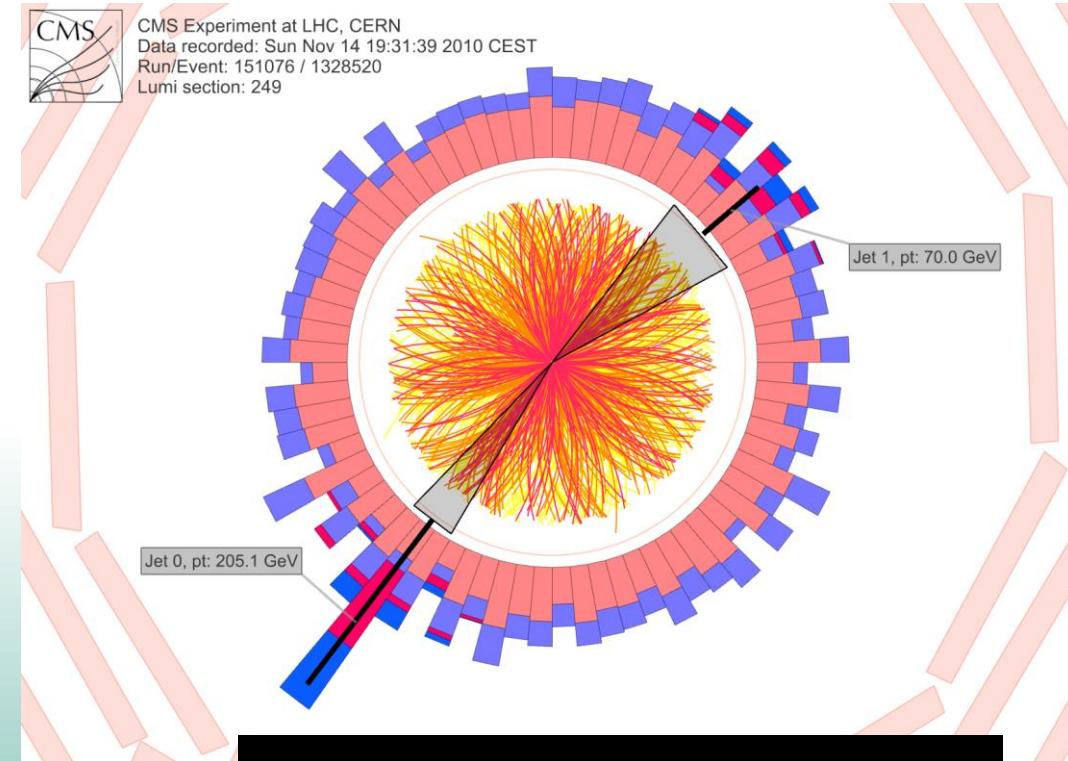
QUANTUM SIMULATION OF OPEN QUANTUM SYSTEMS IN HEAVY-ION COLLISIONS

As written by: *Wibe A. de Jong, Mekena Metcalf, James Mulligan, Mateusz Płoskoń, Felix Ringer, and Xiaojun Yao*

Phys. Rev. D 104, L051501 – Published 7 September 2021

HARD PROBES IN HEAVY-ION COLLISIONS

- Experiments @ RHIC, LHC: create **hot** ($T \approx 150 - 500$ MeV), short-lived ($t \approx 10$ fm/c) QGP
- Jets or heavy quarks:
energy scale \gg QGP temperature
= hard probes
- Markovian limit OK, environment correlation time
 \ll subsystem relaxation time
- Generalization of Schrödinger eq.: **Lindblad eq.**
- Fully field-theoretical description: approximations needed (e.g. semiclassical Boltzmann or Fokker-Planck equations)
- However...
As the size of the subsystem increases → impossible to solve Lindblad on classical computers



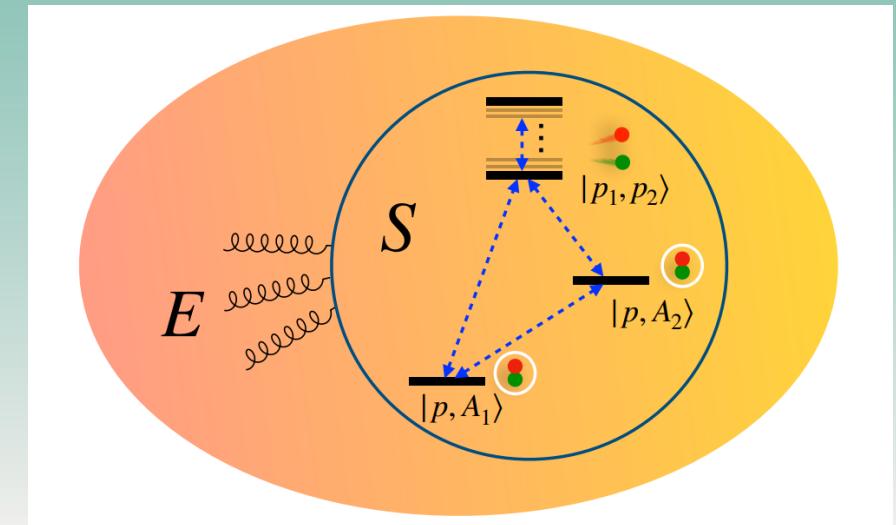
https://www.nsf.gov/news/mm/media/images/PF2297_h.jpg

- Instead of phenomenological, semiclassical calculations: direct quantum simulation, compiled on single- and multi-qubit gates
 - Using NISQ (Noisy Intermediate Scale Quantum devices)
- Can be applied to large size systems, in principle intractable with classical methods
- The Hamiltonian of the full system consisting of the hard probe (subsystem) and the QGP (environment) can be written as:

$$H = H_S + H_E + H_I$$

$$H_S = H_{S0} + H_{S1}$$

- Free + interacting part
- Multi-particle states $|p_1, A_1\rangle \otimes \dots \otimes |p_n, A_n\rangle$



- Evolution: Markovian Lindblad

$$\begin{aligned}\frac{d}{dt} \rho_S(t) = & -i[H_{S1}(t) + H_L, \rho_S(t)] \\ & + \sum_{j=1}^m \left(L_j \rho_S(t) L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho_S(t) \} \right)\end{aligned}$$

with

$$\begin{aligned}\rho^{(\text{int})}(t) &\equiv e^{i(H_{S0}+H_E)t} \rho(t) e^{-i(H_{S0}+H_E)t} \\ H_{S1}^{(\text{int})}(t) &\equiv e^{iH_{S0}t} H_{S1} e^{-iH_{S0}t} \\ H_I^{(\text{int})}(t) &\equiv e^{i(H_{S0}+H_E)t} H_I e^{-i(H_{S0}+H_E)t}.\end{aligned}$$

+assume that the initial density matrix factorizes & the environment density matrix is a thermal state

$$\begin{aligned}\rho(0) &= \rho_S(0) \otimes \rho_E \\ \rho_E &= \frac{e^{-\beta H_E}}{\text{Tr}(e^{-\beta H_E})}\end{aligned}$$

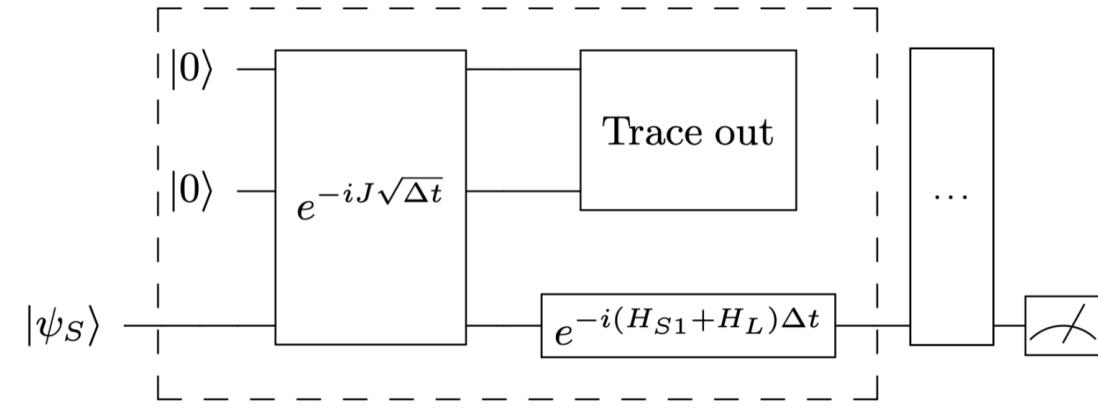
QUANTUM ALGORITHM

- based on the Stinespring dilation theorem, to simulate Lindblad
- Evolution operators J :

$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

& H_S : see fig.

- Simulate on IBM Q → requires a large number of fault-tolerant qubits
 - Toy model for hard probes
 - Special init.



$$H_S = H_{S0} = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[\frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \right]$$

$$H_I = g X \otimes \phi(x=0),$$

RESULTS

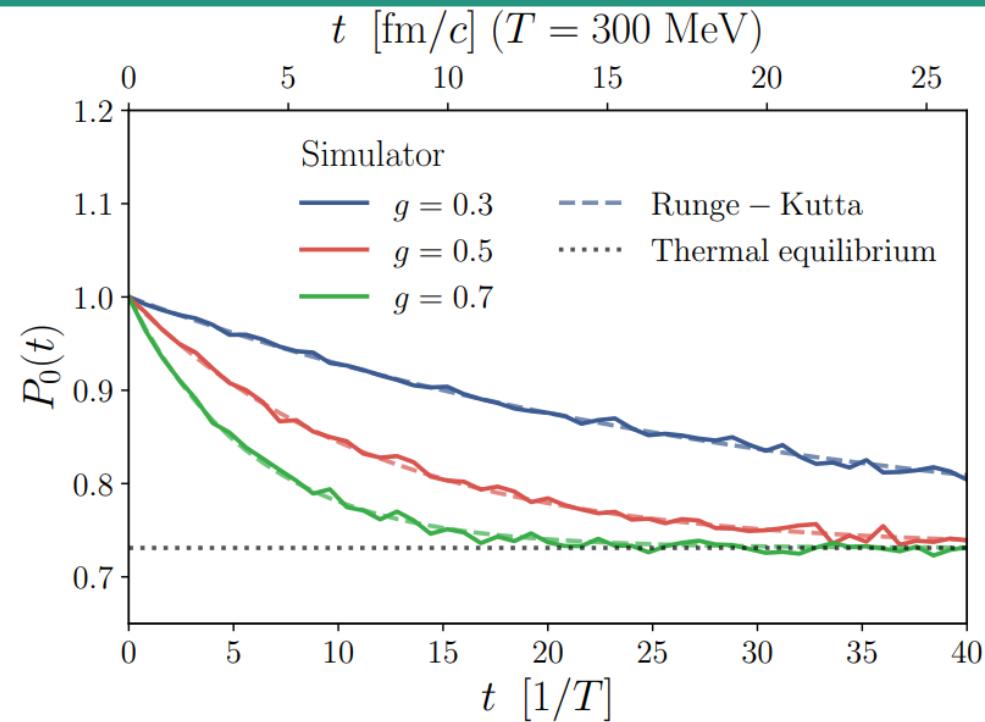


FIG. 3. Simulation of the quantum circuit with $N_{\text{cycle}} = 100$ for various system-environment couplings, along with numerical solution using a 4th order Runge-Kutta method. The upper time axis corresponds to a medium with a temperature of $T = 300 \text{ MeV}$. Each time point in the simulator result consists of 80192 shots (runs).

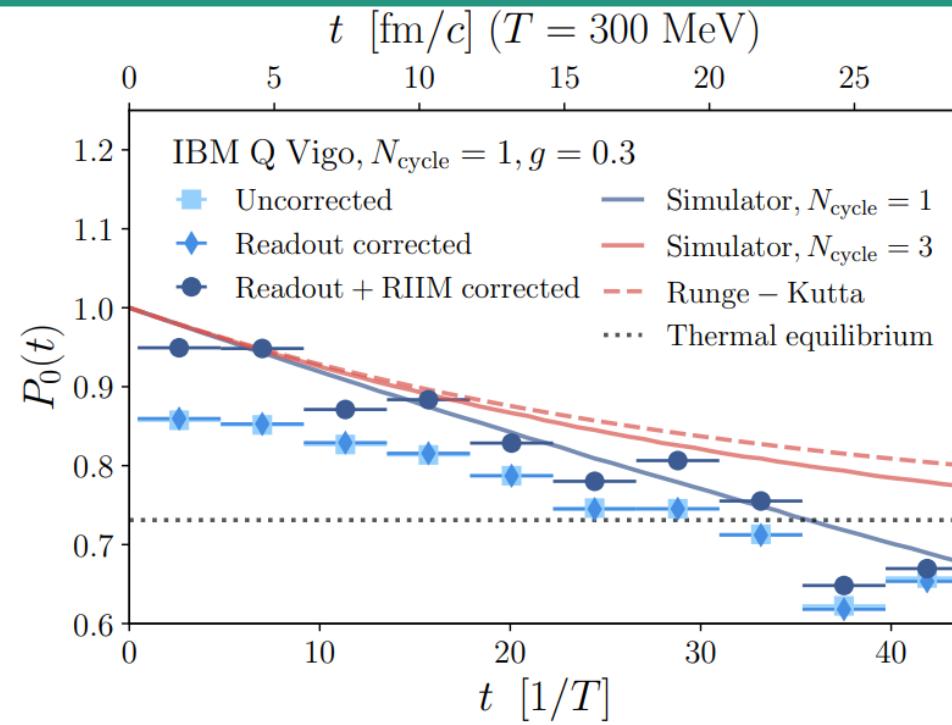


FIG. 4. Results from the IBM Q Vigo device including different error mitigations compared to results from the `qiskit` simulator for $N_{\text{cycle}} = 1$ and $N_{\text{cycle}} = 3$ and the Runge-Kutta method. Higher values of N_{cycle} quickly converge to the result using the Runge-Kutta method. Each time point in the simulator result consists of 800192 shots (runs).

CONCLUSIONS

- used IBM's *qsearch* compiler to construct the quantum circuit
- implemented 2 qubit gate *error mitigation*
- good agreement of the results from the quantum device with the results from the simulator
- *Future:* relevant for various other systems in nuclear and high-energy physics (e.g. studies of Cold Nuclear Matter effects)

THANK YOU FOR YOUR ATTENTION!

SOURCES, REFERENCES

- [1] - Kandala, Abhinav, et al. "Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets." *Nature* 549.7671 (2017): 242-246.
- [2] - DiVincenzo, David P. "The physical implementation of quantum computation." *Fortschritte der Physik: Progress of Physics* 48.9-11 (2000): 771-783.
- [3] Breuer, H.-P.; Petruccione, F. (2007). *The Theory of Open Quantum Systems*. Oxford University Press. p. vii. "Quantum mechanical systems must be considered as open systems"
- [4] von Neumann, John (1927), "Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik", *Göttinger Nachrichten*, 1: 245–272
- [5] Kosloff, Ronnie (2013). "Quantum Thermodynamics: A Dynamical Viewpoint". *Entropy*. 15 (6): 2100–2128. arXiv:1305.2268. Bibcode:2013Entrp..15.2100K. doi:10.3390/e15062100. ISSN 1099-4300.