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AMETEK MATERIALS ANALYSIS DIVISION

Application Note: Variance in counting statistics

Typical question: I am measuring a photopeak where the net area N averages 10,000 counts in 60 seconds. I expect the standard deviation to be √N so 1%, but I did many repeated measurements and I observe 1.6%. What is happening?

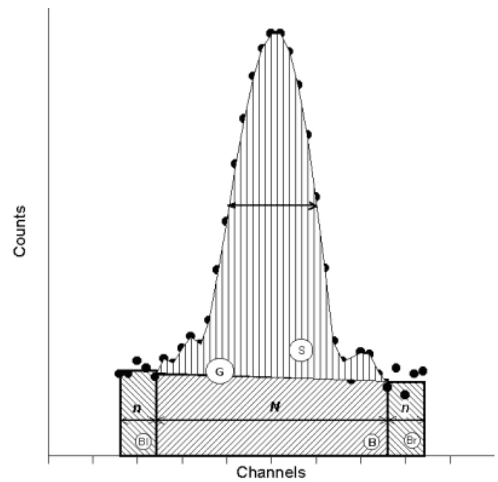
Answer

There are two issues. First, Poisson statistics applies to the directly measured number of counts while the net area is derived. Poisson statistics gives more variance than arises from the net counts alone. One must work through the propagation of the uncertainty for this expression[1]. Second, there are other factors (beyond Poisson statistucs) which add to measurement uncertainty.

Poisson Statistics

The plot on the right sketches a typical photopeak. The total counts in the region of interest (ROI), or gross counts, are denoted *G*. This is directly measured. These counts include the net counts, *N*, from the source and also background counts, *B*. We can write

$$G = N + B$$



The software must estimate and remove the background, B_{est}, to find the estimate of *N*:

$$N_{est} = G - B_{est}$$

and therefore

$$\sigma^2_{N_{est}} = \sigma^2_G + \sigma^2_{B_{est}} = G + \sigma^2_{B_{est}} = N + B + \sigma^2_{B_{est}}$$

There are three interesting points here:

1) It is the gross counts *G* which determine the Poisson statistical limit, $G^{1/2}$. The presence of background counts increases the variance above that due to *N*, the photopeak counts alone.
2) In the limit of large background, *B>>N*, the background counts dominate. The number of counts, *N*, in the photopeak does not matter for *B>>N*!
3) The algorithm used to estimate the background counts impacts the variance and raises the standard deviation above that due to photopeak gross counts alone.

Count rate

It is important to note that it is the counts, not the count rate, which obey Poisson statistics. If we measure 10^4 counts in a second, then the uncertainty is 100 counts, or 1%. If we average over ten seconds, the count rate remains 10^4 cps but the uncertainty is 0.3%.

[1] For a review of uncertainties in counting problems, refer to Chapter 3 of Knoll, G.F., **Radiation detection and measurement**, 4th edition, John Wiley & Sons, 2010.

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Background estimation (1)

In Amptek's DPPMCA[2], the background in the ROI is estimated from small ROIs on either side. We define B_l (B_r) as the total counts in the left (right) ROI. The primary ROI is W channels wide while the small ones are w wide. The background is estimated as

$$B_{est} = \left(B_l + B_r\right)\left(\frac{W}{2w}\right)$$

From this, we can derive that

$$\sigma^2_{B_{est}} = \left(\frac{W}{2w}\right)^2 \left(\sigma^2_{B_l} + \sigma^2_{B_r}\right) = \left(\frac{W}{2w}\right)^2 \left(B_l + B_r\right)$$

It turns out that this can be a significant contributor to σ_N. Note that

$$\sigma_{B_{est}} \propto \frac{\sqrt{B_l + B_r}}{w}$$

and since the B_l and B_r are both roughly proportional to w, $\sigma_{Best} \sim w^{-1/2}$. How you set the parameters in the background estimation algorithm can have a significant impact on the statistical uncertainty in the photopeak counts. The overall uncertainty is

$$\sigma^2_N = \sigma^2_G + \sigma^2_{B_{est}} = G + \left(\frac{W}{2w}\right)^2 \left(B_l + B_r\right) = N + B + \left(\frac{W}{2w}\right)^2 \left(B_l + B_r\right)$$

Background estimation (2)

Consider now another common background estimator: one measures the background counts, G_b, in the absence of a source for some time T_b (not necessarily equal to the actual measurement time T_m). One then subtracts:

$$N_{est} = G - B_{est} = G - G_B\left(\frac{T_m}{T_b}\right)$$

Neglecting uncertainties in the measurement times,

$$\sigma_{B_{est}} = \sqrt{G_B}\left(\frac{T_m}{T_b}\right)$$

so

$$\sigma^2_{N_{est}} = \sigma^2_G + \sigma^2_{B_{est}} = G + G_B\left(\frac{T_m}{T_b}\right)^2 = N + B + G_B\left(\frac{T_m}{T_b}\right)^2$$

If T_m=T_b, i.e. the background is measured for the same duration as the signal, the G_B=B so

$$\sigma^2_N = N + 2B$$

The key for this algorithm is to measure the background for much longer, so counting statistics in this calibration become unimportant.

[2] DPPMCA Help

Additional variations

There are always additional sources of variability or uncertainty, beyond Poisson statistics. For example, in a recent measurement at Amptek, the photopeak counts were measured over three days using a ^{109}Cd radioactive source. The half life is 463 days, so over the 65 hour test, we expect the intensity to decrease to 99.6% of the initial value. Thus there is an additional source,

$$\sigma_G^2 = G + \sigma_{decay}^2$$

We can assume that the decay gives an intensity which decays uniformly from 1.0 to 0.996 of the initial value. A uniform random process over a range Δ (here 0.04 of N_0) leads to a standard deviation of Δ/2√3, so in this case

$$\sigma_{decay}^2 = N_0^2 \left(\frac{1 - 2^{-T_{elaps}/T_{half}}}{12} \right)$$

This example used a radioactive source; its intensity will always change with time, which adds some variability but the variation is highly predictable. Many measurements use X-ray tubes and their stability is far more problematic; the flux can vary with power supply voltage, temperature, even aging. Another source of variability might be a temperature drift: if the temperature of the system drifts with time, then the gain of the processor (which will usually have a non-zero temperature coefficient) also drifts with time. A portion of the Gaussian will drift out of the ROI.

Specific Example

Consider a simple example, recently measured in the lab. A ^{109}Cd and a ^{55}Fe source were positioned about 10 cm in front of the spectrometer, an X-123-SDD. The system was in a fixed geometry, in a chamber at a fixed temperature (23°C), with the detector cooled to 238K (regulating). A spectrum was recorded every minute for approximately three days. The software recorded the net counts, *N*, in the region of interest (ROI) defined around the 22.104 keV photopeak in each spectrum. We computed the standard deviation in the net counts.

The table below shows the results. The average net counts in the photopeak were measured to be *N*=612,711 while we measured σ_N =1,176. Now √N=783, which was a naïve expectation for σ_N. The rows below explain the difference. We estimated the background using method 1 above, and this contributes almost as much to σ_N as does √G. The decay in ^{109}Cd contributes a similar amount. When all three terms are included, agreement is good.

Net counts	612,711	783	Square root of N
		1,250	Expected standard deviation in N
		1,176	Measured standard deviation in N
Gross counts	624,261	790	Square root of G
Background counts	11,550	636	Uncertainty in background estimate
		731	Variance from ^{109}Cd decay