How sensitive is the GammaRad?

This is one of the most commonly asked questions. Sensitivity is very important, arguably the most important performance parameter in many applications of gamma-ray spectroscopy. However, there is a great deal of confusion surrounding sensitivity, for two reasons:

(1) When a user asks about sensitivity, he or she might mean the “intrinsic efficiency”, or the “intrinsic photopeak efficiency”, or the “absolute efficiency”, or the “minimum detectable activity”, and so on. There are many different quantities related to “sensitivity”.

(2) Many of these quantities depend not only on the instrument itself, but also on the measurement techniques, on the analysis software used, on background radiation levels, and so on.

The various definitions of “efficiency” relate the fraction of pulses which are detected to those emitted or incident on the detector. Efficiency may be defined as intrinsic vs absolute, total vs photopeak, etc. These quantities are used to estimate the count rate from a detector in a particular measurement. Efficiency depends on the scintillation material, its geometry, the distance to the source, and the energy of γ-rays.

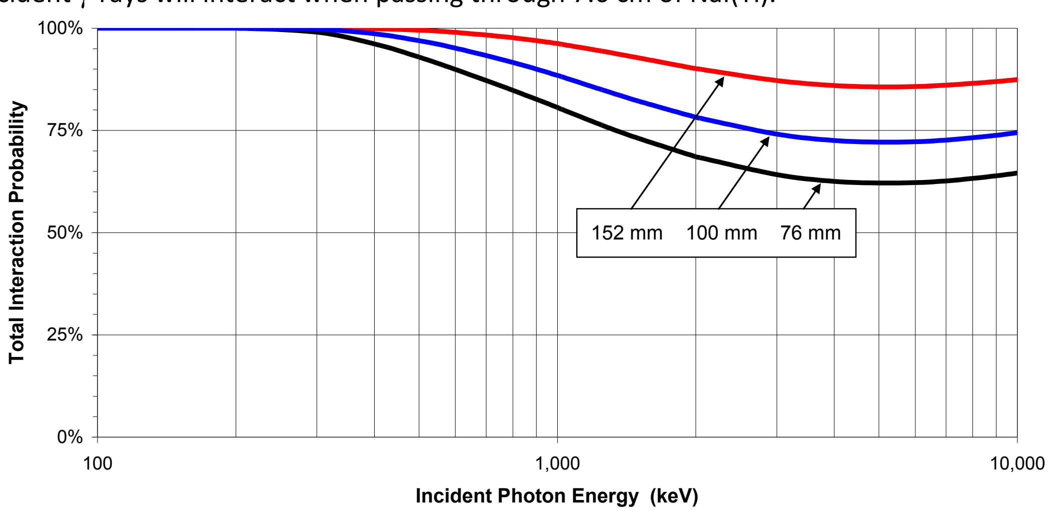
The “minimum detectable activity” answers the question: what is the minimum amount of radioactive material which can be detected with confidence? It is related to the minimum number of counts required to distinguish a source from background. It depends not only on the efficiency of the detector but also on the background radiation level, the signal processing algorithms, measurement time, etc..

This note will briefly define the various parameters used to quantify sensitivity and will show typical values for Amptek’s GammaRad. The information in this note is a summary of information described in more detail in various references. For a general discussion of sensitivity, we recommend the text by Knoll[1] and the on by Gilmore and Hemingway[2]. Most of this discussion is not unique to the GammaRad, but would be applicable to any detector using a scintillator and PMT with similar properties.

1 EFFICIENCY

1.1 INTRINSIC TOTAL EFFICIENCY

The *intrinsic total efficiency* is defined as the ratio of the total number of events which are detected to the total number of γ-ray photons incident on the detector. It is determined by the total attenuation coefficient. The plot below shows the total interaction probability, the probability of any interaction occuring, for γ-rays passing through 7.6, 10.0, and 15.2 cm of NaI(Tl). At 662 keV, for example, about 87% of the incident γ-rays will interact when passing through 7.6 cm of NaI(Tl).

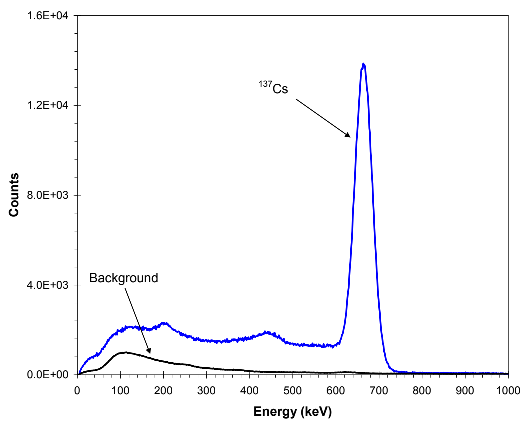


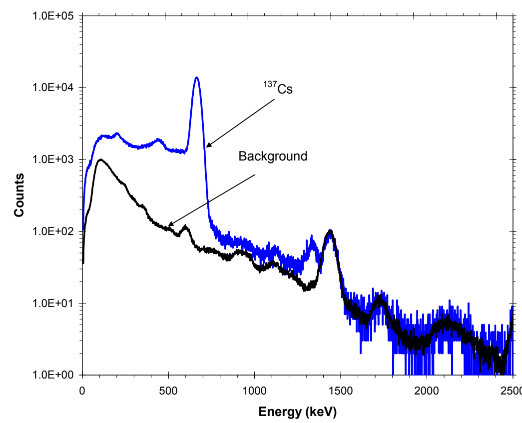
The values in this plot are a good estimate but there are additional effects. First, the scintillator is surrounded by 1.5 mm Al, which attenuates photons below ~30 keV. Second there are geometric effects: if γ-rays are incident along the side of the cylindrical scintillator, then the path length varies with incident location. For sources at a large distance, this is a small correction: for a 7.6 cm diameter cylinder, at 662, the efficiency correction is ~1%. But if the distance to the source is less than 20 times the detector dimension, then the correction may be quite important[3].

Third, it is possible for the signal processing electronics to fail to detect an interacting γ-ray. If the amplitude of the pulse is very small, below the noise threshold of the electronics, it will not be detected, but in the GammaRad the noise threshold is very low so this is usually negligible. Since radioactive decays occur at random times, the electronics may be processing one pulse when a second interaction occurs. These dead time losses are usually negligible at low counting rates but at sufficiently high count rates are important.

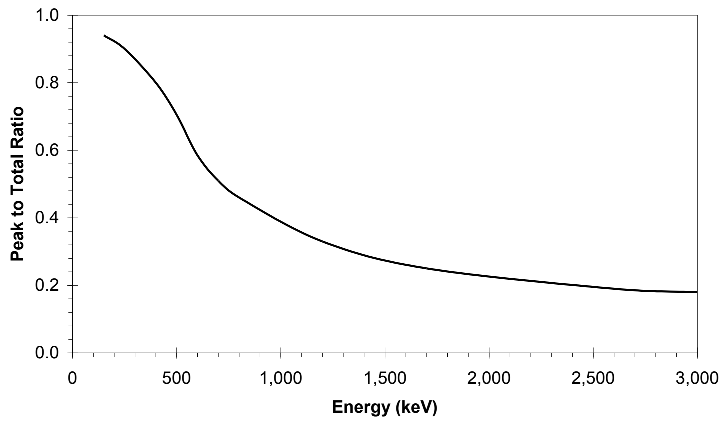
1.2 PHOTOFRACTION

The plots below show a spectrum measured from a ^{137}Cs source and a background spectrum. The plot on the left is a linear plot of the data. Many of the counts occur in a Gaussian peak at 662 keV, which is energy at which the ^{137}Cs emits γ-rays. This is termed the *photopeak*. But there are many counts from the ^{137}Cs at lower energies, arising from scattering events. In these, the incident γ-ray deposits a portion of its energy in an interaction which produces a secondary particle, which may then exit the detector. The plot on the right is a logarithmic plot of the same data. The signal to background ratio is clearly improved if one uses only the photopeak or full energy counts. Photopeak counts are produced by photoelectric interactions or if the incident γ-ray is scattered and the secondary particle is stopped in the detector.





The *photofraction* is defined as the ratio of the number of events which deposit their full energy in the detector, forming the photopeak, to the total number of events which are detected. There is no easy way to calculate it. It is easy to compute the probability of photoelectric interactions: at 662 keV, about 20% of the incident γ-rays undergo a photoelectric interaction in 7.6 cm of NaI(Tl). But in a 7.6x7.6 cm NaI(Tl) detector, the photopeak efficiency is closer to 50%, so most of the photopeak events are not due to photoelectric interactions. The photofraction can be computed using Monte Carlo simulation methods or measured. The figure below shows results which were measured for a 3"x3" cylindrical NaI(Tl) scintillator[4]. The photofraction increases for larger detectors, as more secondaries are captured. It depends on the geometry of the detector and also on the distance to the source.



1.3 INTRINSIC PHOTOPEAK EFFICIENCY

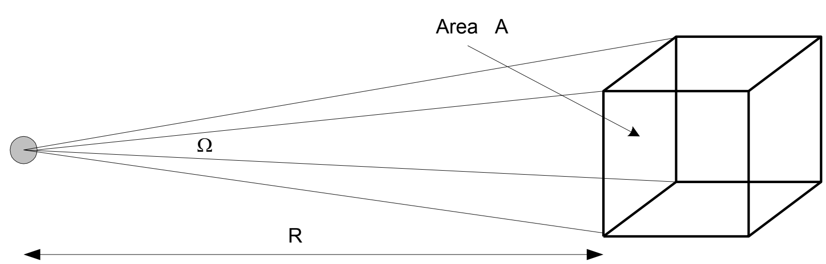
The *intrinsic photopeak efficiency* is the ratio of the number of full energy events to the total number of γ-ray photons incident on the detector. It is the product of the *intrinsic total efficiency* and the *photofraction*. For ^{137}Cs and a 7.6x7.6 cm NaI(Tl), it is (50%)(87%)=43%.

1.4 ABSOLUTE TOTAL EFFICIENCY

The *absolute total efficiency* is defined as the ratio of the total number of events which are detected to the total number of γ-ray photons emitted by the radioactive source. It is the product of intrinsic total efficiency and a geometric factor G_F, which yields the fraction of the emitted γ-rays which are incident on the detector. Consider the geometry sketched below. A source is located a distance R from a detector with area A facing the source. The γ-rays are emitted isotoprically, i.e. with equal probabiliy in all directions, with solid angle 4π steradians. Seen from the source, the detector subtends a solid angle Ω, defined by the area of the detector, which determines the geometric factor. At a large distance, the fractional solid angle is just the area A divided by the area of a sphere with radius R, and this is the fraction of γ-rays incident on the detector. For a 7.6 cm dia detector at 60 cm, the geometric factor is 0.001, as calculated below.

$$G_F = \frac{A}{4\pi R^2}$$

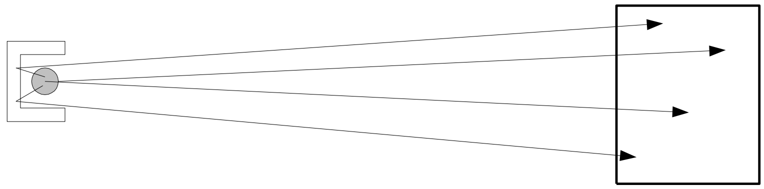
$$\frac{\pi \cdot (3.8 \cdot \mathrm{cm})^2}{4 \cdot \pi \cdot (60 \cdot \mathrm{cm})^2} = 1.003 \times 10^{-3}$$



The simple formulas only holds if the separation between the source and detector is large. If we consider the limit of small separation, the detector subtends half the solid angle, and this does not change even if the source is moved slightly away from the detector. For intermediate values, analytical approximations have been computed for a variety of different geometries[5].

Note that the absolute efficiency is defined in terms of the number of γ-rays emitted by the source, rather than by the number of radioactive decays. Not every decay is accompanied by the emission of a γ-ray. For example, in ^{137}Cs the 662 keV γ-ray is emitted in 85% of the decays.

Note also that the absolute efficiency assumes there is no attenuation between the source and the detector and no scattering from adjacent materials. If the source is shielded, or is embedded in a matrix which attenuates the γ-rays, this will affect the count rate. Even more subtle is the effect of surrounding materials. Consider the geometry below, where the source is located in a collimator which does not restrict the solid angle impinging on the detector. Some photons from the source will be emitted away from the detector, but then will backscatter from the collimator and thus add to the signal. Computing or measuring absolute count rates is quite challenging.



1.5 Absolute Photopeak Efficiency

The *absolute photopeak efficiency* is defined as the ratio of to full energy events to the total number of γ-ray photons emitted by the radioactive source. It is the product of absolute total efficiency and photofraction.

1.6 Relative Efficiency

The *relative efficiency* is defined as the ratio of the efficiency of a detector to that of a 3"x3" NaI(Tl) scintillator at 1332 keV, in the same geometry and under the same conditions. This parameter can be confusing, since it is expressed in the same units as the other efficiencies. The relative efficiency may certainly be >100%, which adds further confusion.

2 SAMPLE COUNT RATE CALCULATION

The worksheet below shows an example count rate calculation for a 1 µCi ^{137}Cs source located 60 cm in front of a cylindrical NaI(Tl) scintillator, 7.6 cm in diameter by 7.6 cm long. The calculation was carried out in MathCad but can easily be carried out using other tools.

The activity of the source is S. In this example, we assume 1 µCi, or 37 kBq, which corresponds to 3.7x10^4 disintegrations per second.

$$S := 10^{-6}\cdot \mathrm{Ci} \qquad S = 3.7\times 10^{4}\cdot \mathrm{Bq} \qquad S = 3.7\times 10^{4}\,\frac{1}{\mathrm{s}}$$

The fraction of disintegration which emit a γ-ray is B_R, the branching ratio. For ^{137}Cs, B_R=85%, so there are 3.1x10^4 γ-rays per second emitted by the source, isotropically..

$$B_R := 0.85 \qquad S\cdot B_R = 3.145\times 10^{4}\,\frac{1}{\mathrm{s}}$$

The geometric factor GF is the fraction of emitted γ-rays which are incident on the detector. Here we assume a 7.6x7.6 cm NaI(Tl), where the source faces the circular end. Only one out of a thousand of the emitted γ-rays is incident on the detector.

$$G_F := \frac{\pi\cdot(3.8\cdot \mathrm{cm})^2}{4\cdot\pi\,(60\cdot \mathrm{cm})^2} \qquad G_F = 1.003\times 10^{-3}$$

The rate of γ-rays incident on the detector is R_{inc}. In this example, it is 31 sec^{-1}.

$$R_{inc} := S\cdot B_R\cdot G_F \qquad R_{inc} = 31.537\,\frac{1}{\mathrm{s}}$$

The total intrinsic efficiency was shown to be 87%, and the photofraction about 50%. From these we can compute the total rate of interactions R_{tot}, and the photopeak rate, R_{peak}.

$$\varepsilon_{intrinsic} := 0.87$$

$$\varepsilon_{photofraction} := 0.5$$

$$R_{tot} := R_{inc}\cdot\varepsilon_{intrinsic}$$

$$R_{peak} := R_{tot}\cdot\varepsilon_{photofraction}$$

For the example here we compute a total rate of 27 sec^{-1} and the photopeak rate of 14 sec^{-1}.

$$R_{tot} = 27.438\,\frac{1}{\mathrm{s}}$$

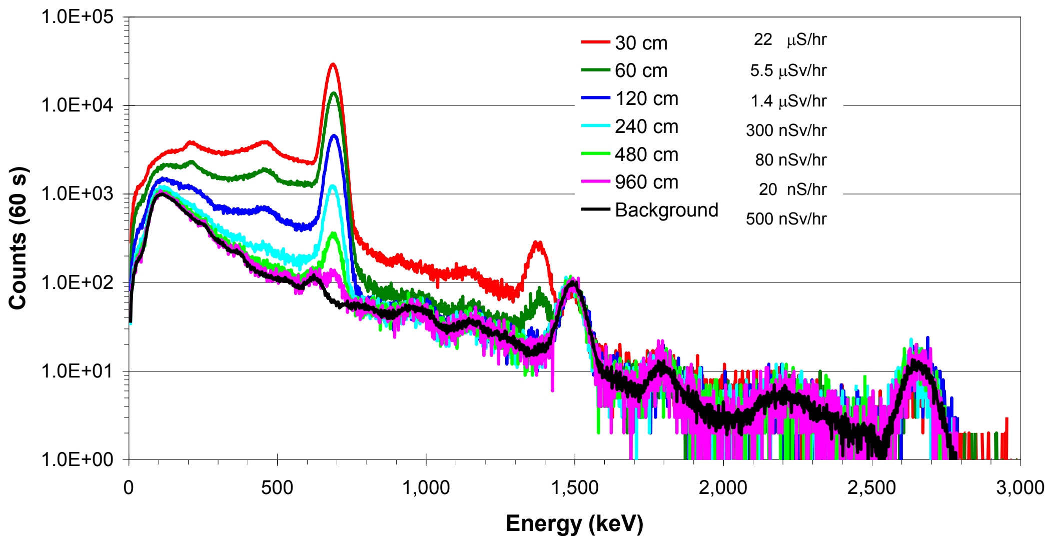
$$R_{peak} = 13.719\,\frac{1}{\mathrm{s}}$$

For some applications, knowing the count rate might be sufficient. However, in many real-world applications, the question is: can we distinguish these count rates from background? In our laboratory at Amptek, we typically find background count rates of 300 sec^{-1}. The background count rate in the 662 keV peak is roughly 7 sec^{-1}. All of these counts are subject to statistical fluctuations.

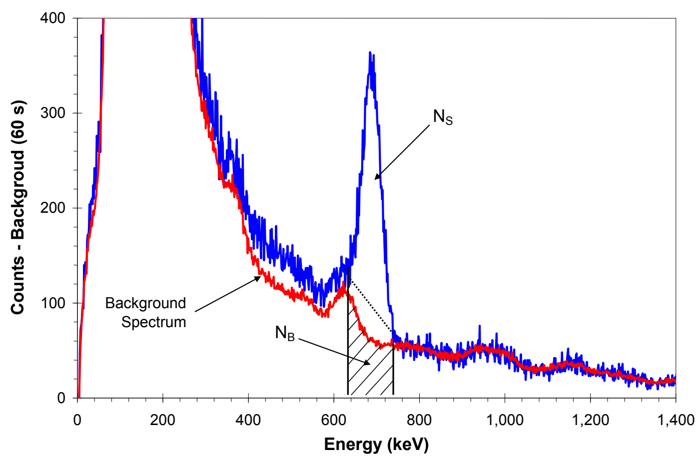
Can we confidently detect the presence of this source? With what statistical confidence? How long must we record data to obtain the necessary confidence? The next section will address these questions.

3 DETECTION LIMITS

The plot below shows spectra measured from an 80 μCi (2.9 MBq) ^{137}Cs source at distances from 30 cm to 10 m, using a large volume scintillator (10x10x40 cm^3 NaI(Tl)). Each spectrum was measured for 60 seconds. Also shown is the background spectrum, measured for 10 minutes and averaged. The dose rate from the background and from the source at each distance is also shown.



The background radiation level was 500 nSv/hr, with total count rate of almost 2,970 sec^{-1}. At 10 m, the ^{137}Cs source added only 20 nSv/hr and 70 sec^{-1}. The signal was much lower than the background. But in the spectrum, one can clearly see the additional counts in the 662 keV photopeak. Looking only at the photopeak, the background rate is 165 sec^{-1}. Even in the photopeak the signal is less than background, yet we can clearly distinguish the signal. So the question is: how well can we distinguish a source from background and random statistical fluctuations? How weak a source can be detected?



The figure above illustrates a typical measurement. We define a region of interest (ROI) within the spectrum, marked by the vertical lines, 620 to 670 keV here. N_B is the number of counts within the ROI due to background, the total area below the red curve. N_S is the number of counts within the ROI due to the signal, the total area between the red and blue curves, sometimes called net counts. N_T is the total number of counts within the ROI, sometimes called gross counts:

$$N_S = N_T - N_B$$

If there were no fluctuations in the counts or other source of uncertainty, then if N_S were greater than zero, we would conclude that a source was present, and if N_S were zero, we would conclude no source is present. But there will always be statistical fluctuations in the counts, due to the random nature of radioactive decays, and there are likely to be additional sources of uncertainty and fluctuation in a real instrument. We therefore must require that N_S be greater than zero by its uncertainty times factor k, which determines the confidence:

$$N_S \geq k\left(\sigma_{N_T}\right)$$

We have

$$\sigma_{N_S}^2 = \sigma_{N_T}^2 + \sigma_{N_B}^2$$

and

$$\sigma_{N_T}^2 = N_T = N_S + N_B$$

so

$$\sigma_{N_S}^2 = \left(N_S + N_B\right) + \sigma_{N_B}^2 \qquad \Rightarrow \qquad N_S^2 \geq k^2\left(N_S + N_B + \sigma_{N_B}^2\right)$$

This implies that N_S must exceed a value which depends on the background rate and on the uncertainty in the estimate of the background rate. There is some lower limit on the total counts, or for a fixed measurement time, on the count rate. For a given absolute efficiency, this implies a minimum activity.

The σ_{NB}^2 term arises because we do not directly measure the background in the ROI, we estimate it from something else. This term represents the uncertainty in the background estimate and so depends on the algorithm used to estimated background. In the plot above, we used a prior measurement of the background to estimate N_B, but this is not always possible. A common alternative is to estimate it from the spectrum itself, for example using the thin gray line drawn across the ROI. The measured counts in channels just outside the ROI are then used to estimate N_B. These two algorithms will yield different values for σ_{NB}^2.

In this example, we used the counts in an ROI around the photopeak to estimate N_T and N_S. One could also use all the counts in the spectrum or various other algorithms. Sometimes one knows the isotopes of interest before the measurement, and so has known ROIs, while in other cases one is trying to find unknown peaks and so must determine the ROIs from the measured spectrum. The spectral analysis algorithms are critical in determining the MDA.

3.1 Critical and Detection Limits

One needs to compare the net counts, N_S, to a threshold to determine if a source is present or not, typically with 95% confidence. The *critical limit* L_C is defined so that, if $N_S<L_C$, we may conclude that no source is present with a false-positive rate no larger than 5%. The *detection limit* L_D is defined so that, if $N_S>L_D$, we may conclude that a source is present with a false negative rate no larger than 5%. It can be shown[6] that

$$L_C = 2.326\ \sigma_{N_B}$$

$$L_D = 4.653\ \sigma_{N_B} + 2.706$$

It may seem suprising that the two limits are different, but if we had a source which gave exactly L_C, then half the time the counts would be below L_C and half the time below, so we only have 50% confidence. To give 95% confidence, L_D must be greater than L_C.

3.2 MINIMUM DETECTABLE ACTIVITY

The *minimum detectable activity* is defined as the minimum amount of radioactive material necessary to yield L_D. We can rewrite this using the terms from the efficiency: S_{MDA} is the minimum detectable activity, T is the measurement time, B_R is the γ-ray yield per disintegration, G_F is the geometric factor, $\varepsilon_{intrinsic}$ is the intrinsic efficiency, and $\varepsilon_{fraction}$ is the fraction of interactions which are summed in the algorithm (this may be the photopeak, all of the counts, or something else). This gives

$$
\begin{aligned}
L_D &= T\square_{meas} \\
&= T\left(S_{MDA} B_R G_F\right)\left(\varepsilon_{\mathrm{int}rinsic}\varepsilon_{fraction}\right)
\end{aligned}
$$

Rearranging this, and noting that ($G_F\ \varepsilon_{intrinsic}\ \varepsilon_{fraction}$) is the absolute photopeak efficiency, yields

$$S_{MDA} = \frac{L_D}{B_R\left(G_F \varepsilon_{\mathrm{int}rinsic}\varepsilon_{fraction}\right)T} = \frac{4.653\ \sigma_{N_B} + 2.706}{B_R\left(G_F \varepsilon_{\mathrm{int}rinsic}\varepsilon_{fraction}\right)T}$$

The σ_{NB} is a strong function of the intensity and spectrum of the background radiation. In many cases, it may be a function of the measurement time. The absolute efficiency is a strong function of distance to the source and of energy. So the MDA can only be properly defined under limited conditions. One cannot state that “the MDA is 1 μCi”. Instead, one must define the energy of the source, the distance to the source, and the background radiation level to properly compare the MDA of two systems.

4 SAMPLE MDA CALCULATION

Consider an example, using the efficiencies and counting rates computed above, if the background count rate in the photopeak is 16 sec^{-1}. Further assume that we characterize the background by a 600 second measurement and then average the result. This leads to a measurement of 9600±98 background counts in the photopeak, for a background rate of 16±0.16 sec^{-1}. In a 30 second measurement, the uncertainty in the background correction will be 4.9 cts.

$$R_{back} := 16 \cdot \frac{1}{s}$$

$$N_{backgnd}(T_{back}) := R_{back} \cdot T_{back} \qquad N_{backgnd}(600 \cdot sec) = 9.6 \times 10^{3}$$

$$\sigma_{backgnd}(T_{back}) := \sqrt{N_{backgnd}(T_{back})} \qquad \sigma_{backgnd}(600 \cdot sec) = 97.98$$

$$\frac{\sigma_{backgnd}(600 \cdot sec)}{600 \cdot sec} = 0.163 \frac{1}{s}$$

$$\sigma_{back}(T_{meas}, T_{back}) := \sigma_{backgnd}(T_{back}) \cdot \frac{T_{meas}}{T_{back}} \qquad \sigma_{back}(30 \cdot sec, 600 \cdot sec) = 4.899$$

Now assume that we measure a 1 µCi ^{137}Cs source at a distance of 60 cm from a 3.8x3.8 cm NaI(Tl) scintillator. From the efficiency calculations above, this yields a photopeak rate of 13.8 sec^{-1}. If we integrate for 30 seconds, then we obtain 412 cts from the source, 480 background counts, and 892 counts gross in the photopeak.

$$N_{peak}(T) := T \cdot R_{peak} \qquad N_{peak}(30 \cdot sec) = 411.563$$

$$N_{back}(T) := T \cdot R_{back} \qquad N_{back}(30 \cdot sec) = 480$$

$$N_{gross}(T) := N_{back}(T) + N_{peak}(T) \qquad N_{gross}(30 \cdot sec) = 891.563$$

If there were no background, then the uncertainty in the peak counts would be 20. With the background, and with the uncertainty in the background removal, the uncertainty in the net counts is 30. If we use a shorter background measurement, e.g. 30 sec, this increases.

$$\sigma_{peak}(T) := \sqrt{N_{peak}(T)} \qquad \sigma_{peak}(30 \cdot sec) = 20.287$$

$$\sigma_{net}(T_{meas}, T_{back}) := \sqrt{N_{peak}(T_{meas}) + N_{back}(T_{meas}) + \left(\sigma_{back}(T_{meas}, T_{back})\right)^2}$$

$$\sigma_{net}(30 \cdot sec, 600 \cdot sec) = 30.258 \qquad \sigma_{net}(30 \cdot sec, 30 \cdot sec) = 37.035$$

We can compute the MDA for the geometry above, using the absolute photopeak efficiency computed earlier. For a 30 second measurement at 60 cm from 3.8x3.8 cm NaI(Tl), in a background of 16 sec^{-1} which has been measured for 600 sec, the MDA is 62 nCi.

$$MDA(T_{meas}, T_{back}) := \frac{4.653 \cdot \sigma_{back}(T_{meas}, T_{back}) + 2.706}{T_{meas} \cdot B_R \cdot (G_F \cdot \varepsilon_{intrinsic} \cdot \varepsilon_{photofraction})}$$

$$MDA(30 \cdot sec, 600 \cdot sec) = 6.196 \times 10^{-8} \, Ci$$

REFERENCES

[1] G. Knoll, *Radiation detection and measurement*, 3rd edition, John Wiley & Sons, Inc., 2000.

The topics of efficiency and MDA are covered on pp 116-119 and 94-96.

[2] G. Gilmore, J.D. Hemingway, *Practical gamma-ray spectrometry*, John Wiley & Sons, 1995

The topics of efficiency and MDA are covered on 119-125 and 137-143.

[3] A.A. Mowlavi, R.I. Najafabadi, R.K. Faygh, "Calculation of intrinsic efficieny of NaI(Tl) detector using MCNP code", *Int. Journ. Pure Appl. Phys*., Vol 1, No 2 (2005), pp 129-136.

[4] From Figure 1 of "Efficiency calculations for selected scintillators", a technical note from Bicron, Inc.

[5] N. Tsoulfanidas, *Measurement and Detection of Radiation*, Hemisphere publishing, 1983, pp 248-258.

[6] Derivations are found in Knoll and in Gilmore and Hemingway, pages listed above.