Experimental methods in particle physics

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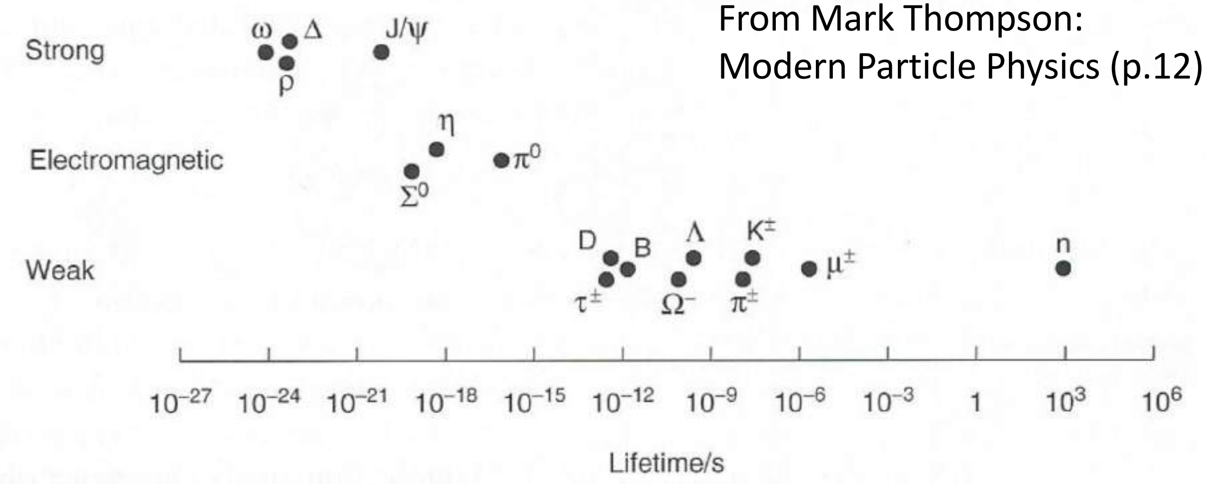
Webpage of lecture:
http://atomfizika.elte.hu/rfkm/rfkm2019.html

Particles' passage through matter
Detectors

H.-C. Schultz-Coulon, Detector lectures:http://www.kip.uni-heidelberg.de/~coulon/Lectures/Detectors/
Claus Grupen, Physics of Particle Detection: http://arxiv.org/pdf/physics/9906063v1.pdf
PDG: http://pdg.lbl.gov/2014/reviews/rpp2014-rev-passage-particles-matter.pdf
ESIPAP 2015: https://indico.cern.ch/event/294651/other-view?view=standard

Particle detection

- Observation of the passage of particles, measurement of their momentum, energy, identification of their type (mass, charge, spin, magnetic moment, lifetime, interactions)
- Stable particles: e, p [uud], γ, ν
- All other particles decay after s = $\gamma v \tau$ distance (v: velocity, $\gamma = 1 / \sqrt{(1-\beta^2)}$: Lorentz-factor, τ: mean lifetime in the rest frame of the particle)
- Relativistic particles with lifetime $\tau \gtrsim 10^{-10}$ s (ex. μ, n [ddu], $\pi^{\pm}$ [$u\bar{d}/d\bar{u}$], $K^{\pm}$ [$u\bar{s}/s\bar{u}$]) travel a few meters in the detector: directly observables
- Short lifetime particles decay before travelling a significant distance, only their decay products are detectable
- **The experimental methods to detect and identify particles relies on the nature of their interaction with matter**



From Mark Thompson: Modern Particle Physics (p.12)

The lifetimes of a number of common hadronic states grouped into the type of decay. Also shown are the lifetimes of the muon and tau-lepton, both of which decay weakly.

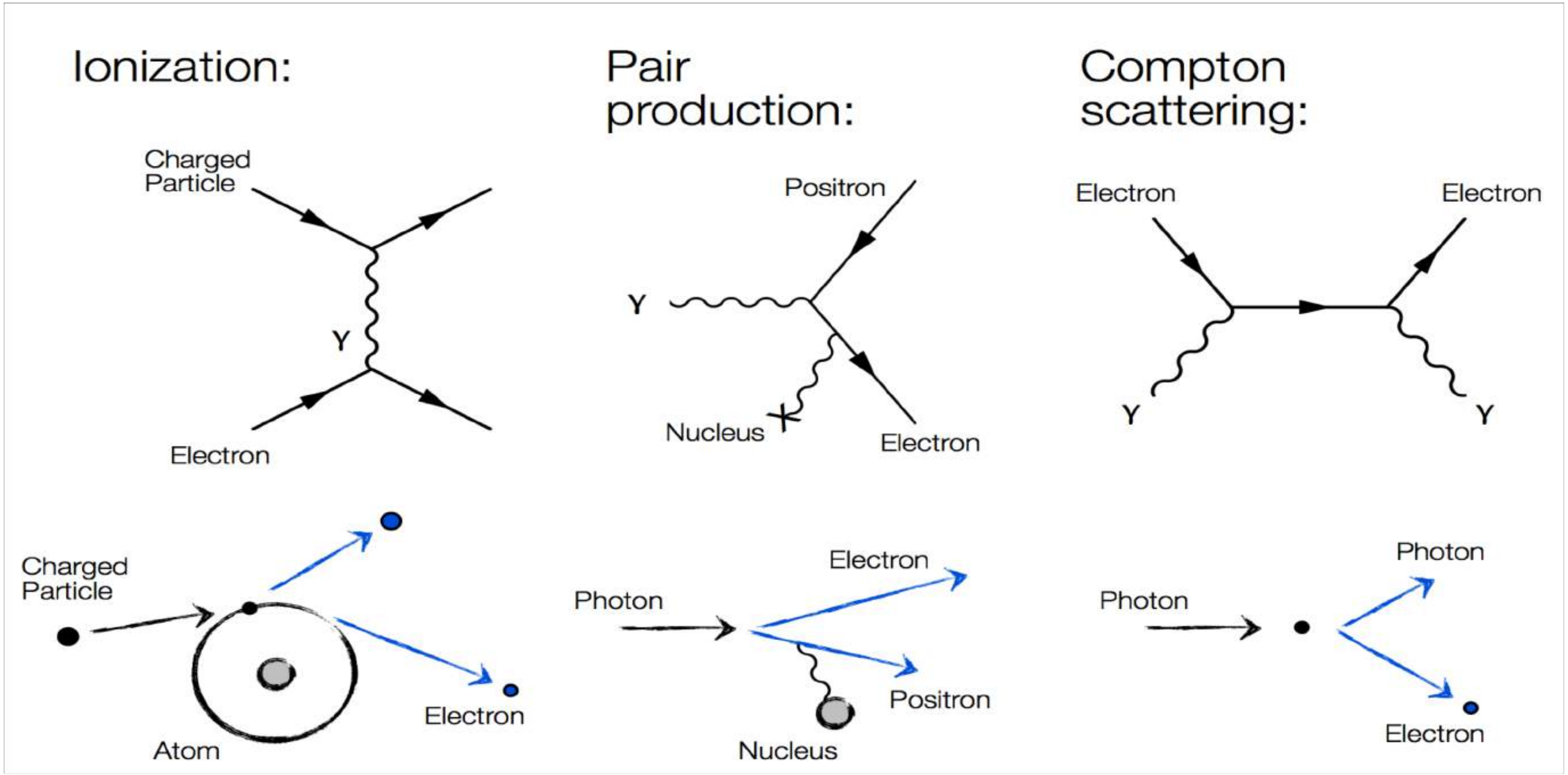
How to detect particles?

- In order to detect particles they need to interact with the detector material and loose energy in it in a measureable way (signal)
- **Particle detection is based on the energy loss of particles in the material they travel through**

- **Charged particles:** ionisation, bremsstrahlung, Cherenkov radiation, transition radiation… (multiple interactions)
- **Photons:** photoelectric effect, Compton scattering, e^+e^- pair-production (single interaction)
- **Hadrons:** Nuclear interactions
- **Neutrinos:** Weak interaction

- **Particle identification: mass, charge, spin… other quantum numbers**

Interactions (a few examples)



Photon stops to exist

Charged particle interacts with atomic electron and continues his travel

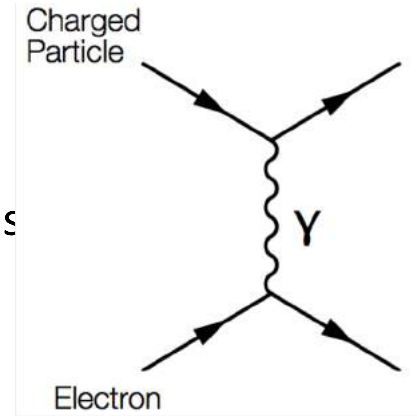
Creates an e+e- pair

Photon is absorbed by electron, a new photons is emitted

Interaction of particles and matter

- **Electromagnetic interactions of charged particles**
- Electromagnetic interactions of photons
- Strong interactions of charged and neutral hadrons

Interactions of charged particles: ionisation



- Relativistic particles interact electromagnetically with the atomic electrons in the medium and they loose energy
- Dominant interaction: elastic scattering with electrons
- Ionisation energy loss in unit path length for unit density: Bethe-Bloch formula (for heavy particles $M \gg m_e$)

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad [\cdot\varrho]$$

- **Particle** charge: z
- … velocity: $\beta = v / c$
- … Lorentz factor: $\gamma = (1 - \beta^2)^{-1/2}$
- $\beta\gamma = p / (m \cdot c)$
- Atomic number of **medium**: Z
- … atomic mass: A
- Mean excitation energy in the medium: $I \sim 10 \cdot Z$ eV
- Maximal energy transfer in a collision: W_{max} [MeV] depends on the particle mass and velocity

$m_e c^2$	electron mass × c^2	0.510 998 928(11) MeV
r_e	classical electron radius $e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 3267(27) fm
N_A	Avogadro's number	$6.022\,141\,29(27) \times 10^{23}$ mol^{-1}
z	charge number of incident particle	
Z	atomic number of absorber	
A	atomic mass of absorber	g mol^{-1}
K	$4\pi N_A r_e^2 m_e c^2$	0.307 075 MeV mol^{-1} cm^2
I	mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	density effect correction to ionization energy loss	

- The above formula is valid with a few % precision for $0.1 < \beta\gamma < 1000$ and for materials with medium Z

Maximal energy transfer

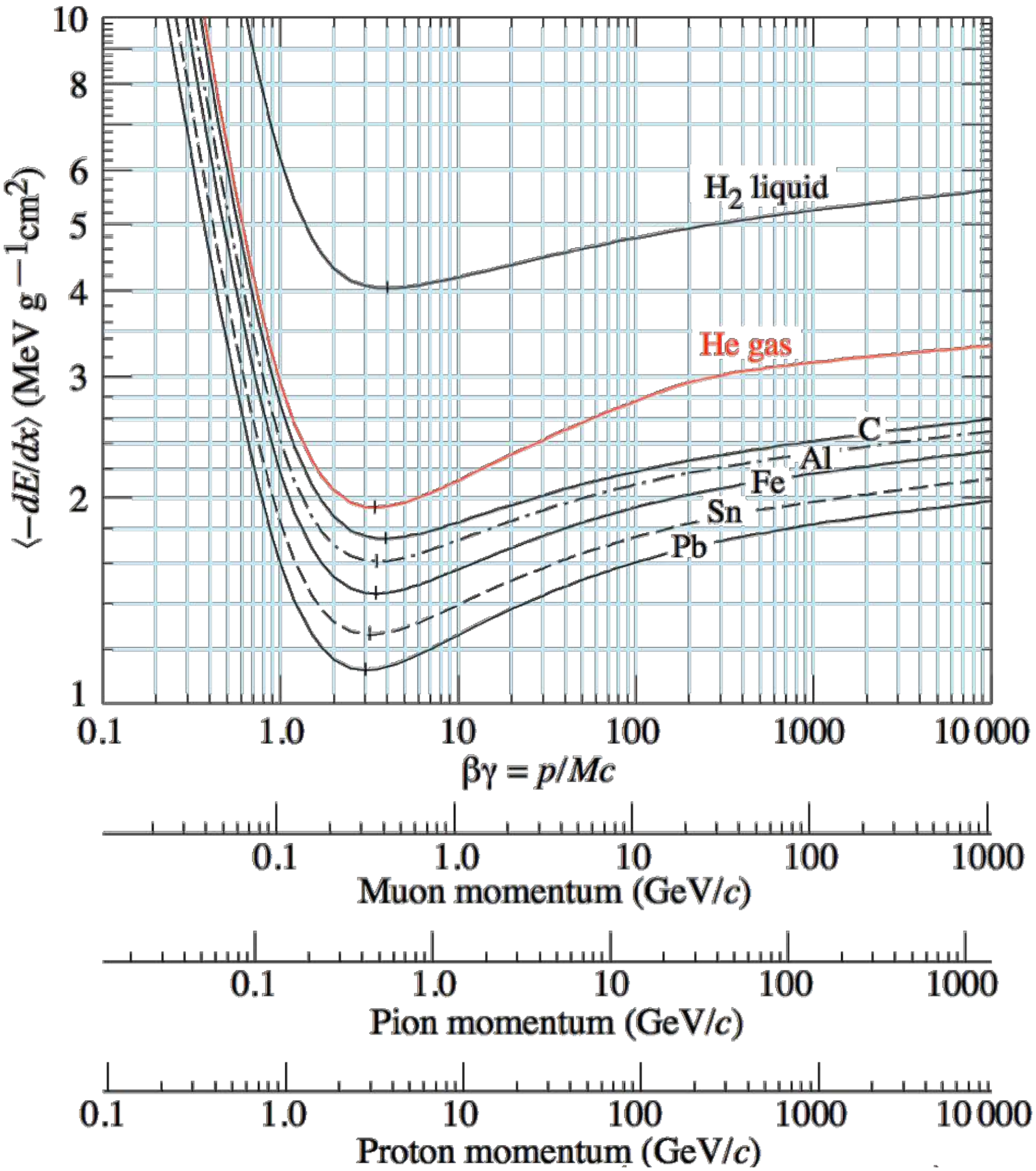
32.2.2. ***Maximum energy transfer in a single collision*** : For a particle with mass M,

$$W_{\max} = \frac{2m_e c^2 \beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} . \qquad (32.4)$$

In older references [2,8] the "low-energy" approximation $W_{\max} = 2m_e c^2 \beta^2\gamma^2$, valid for $2\gamma m_e \ll M$, is often implicit. For a pion in copper, the error thus introduced into dE/dx is greater than 6% at 100 GeV. For $2\gamma m_e \gg M$, $W_{\max} = Mc^2 \beta^2\gamma$.

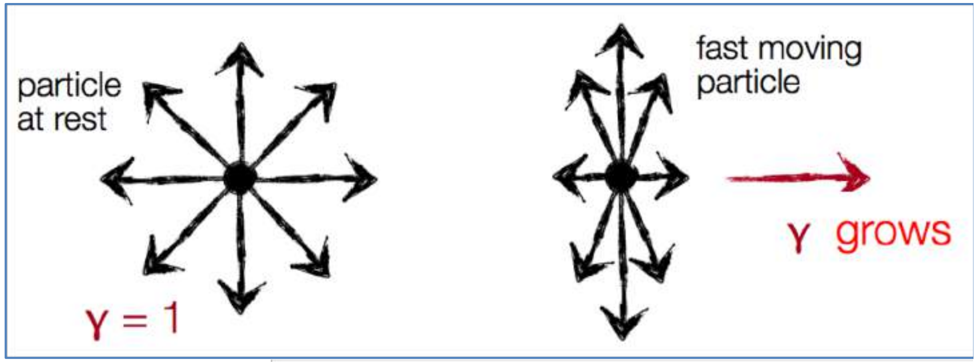
Ionisation energy loss

- Ionisation energy loss does not depend strongly on the type of absorber (<dE/dx> ~ Z/A ~ 1/2), except its dependence on the density
- Minimal ionisation energy loss at $\beta\gamma \approx$ 3-4 approx. 1-2 MeV/(g cm^{-2}) → minimum ionising particles (MIPs)
- At small velocities <dE/dx> ~ 1/β^2 grows steeply:
  slower particles fell longer the electric field of atomic electrons



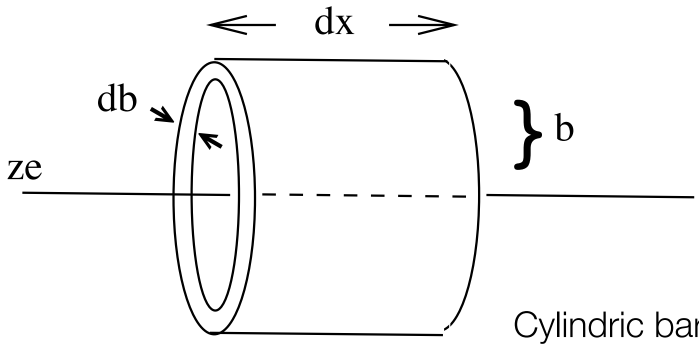
Ionisation energy loss

- Highly relativistic particles (v≈c) $\beta\gamma>4$:
  $\langle dE/dx \rangle \sim \ln(\beta\gamma)$ "relativistic rise"
  - High energy particle: The electric field in the direction perpendicular to the velocity gets stronger due
    to Lorentz transformation: $E_\perp \rightarrow \gamma \cdot E_\perp$
  - The interaction cross-section grows



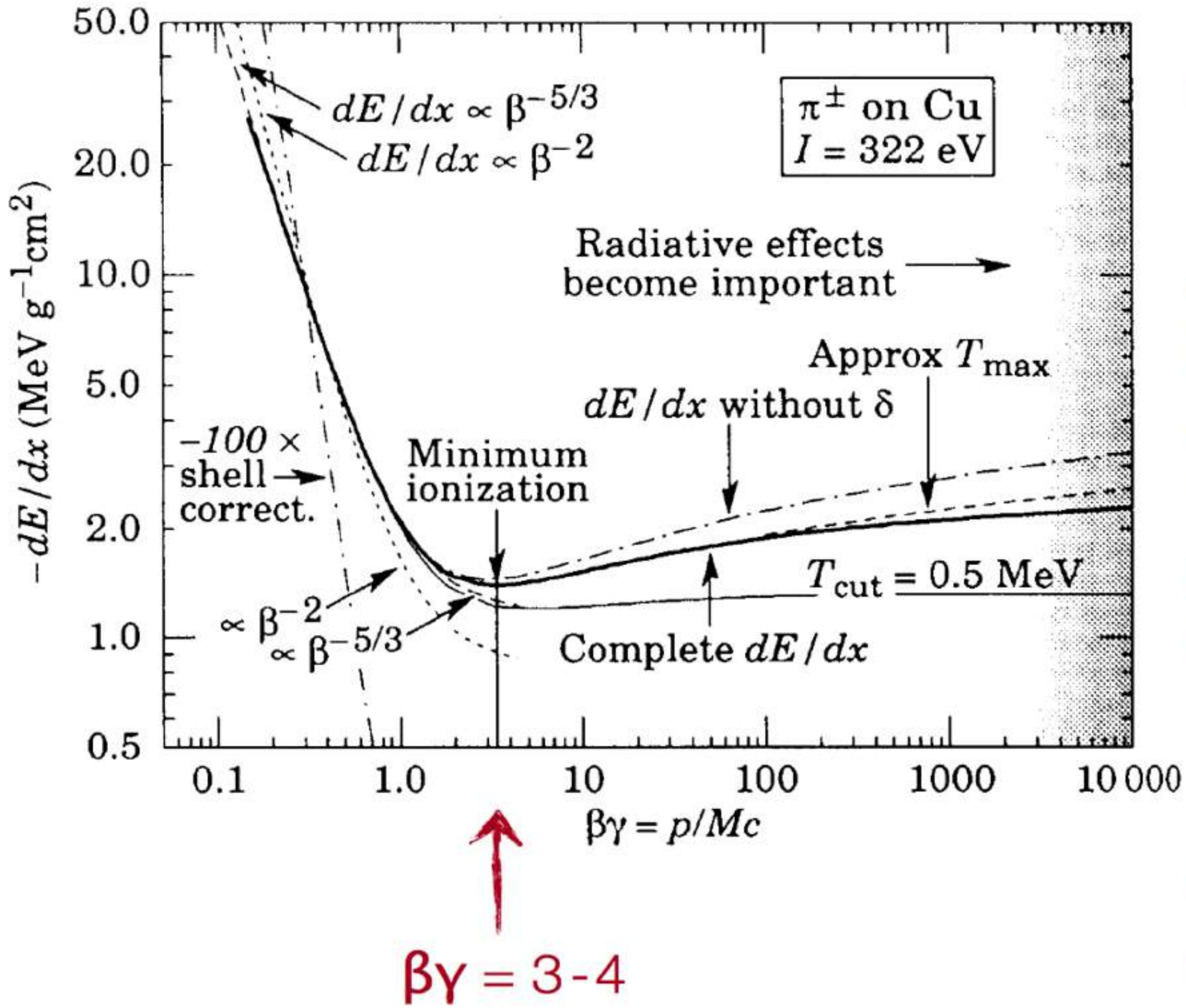
- Corrections
  - Low energy: shell corrections (usually small)
    - When the speed of the incident particle is close to the orbit speed of the electron
    - The assumption that the atomic electron is in rest is broken
    - Electron capture is possible
  - High energy: density correction -C/Z
    - Density dependent polarisation effect:
      The long-range contribution is screened by the electric field off the particle path
    - At large γ: the "range" of the electric field grows, b_{max} larger

$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln\beta\gamma - 1/2$$



Cylindric barrel with N_e electrons

Pion energy loss in copper



Minimum ionizing particles (MIP): $\beta\gamma$ = 3-4

dE/dx falls ~ β^{-2}; kinematic factor
[precise dependence: ~ $\beta^{-5/3}$]

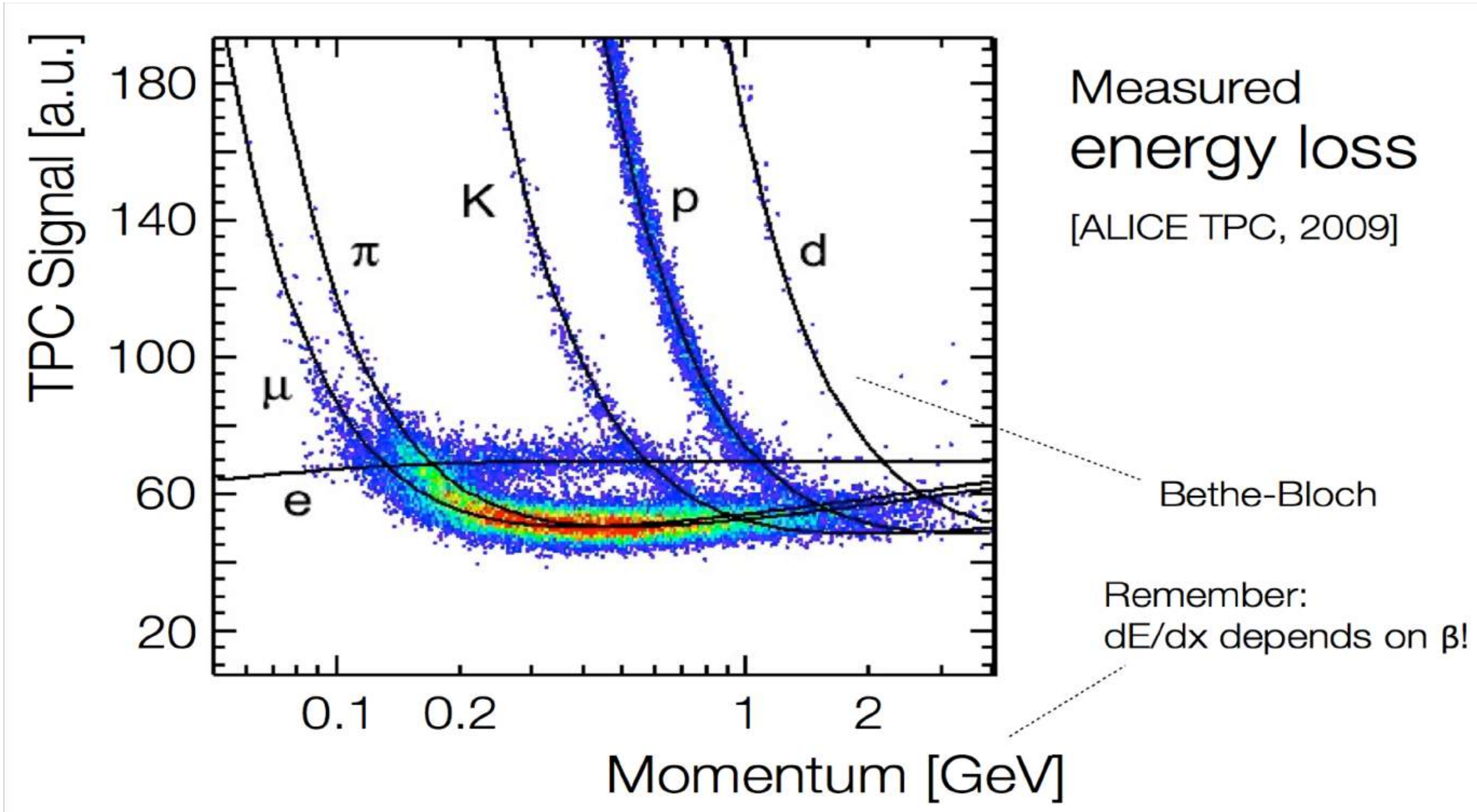
dE/dx rises ~ $\ln(\beta\gamma)^2$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large ($\beta\gamma$) due to density effect (correction δ)
[polarization of medium]

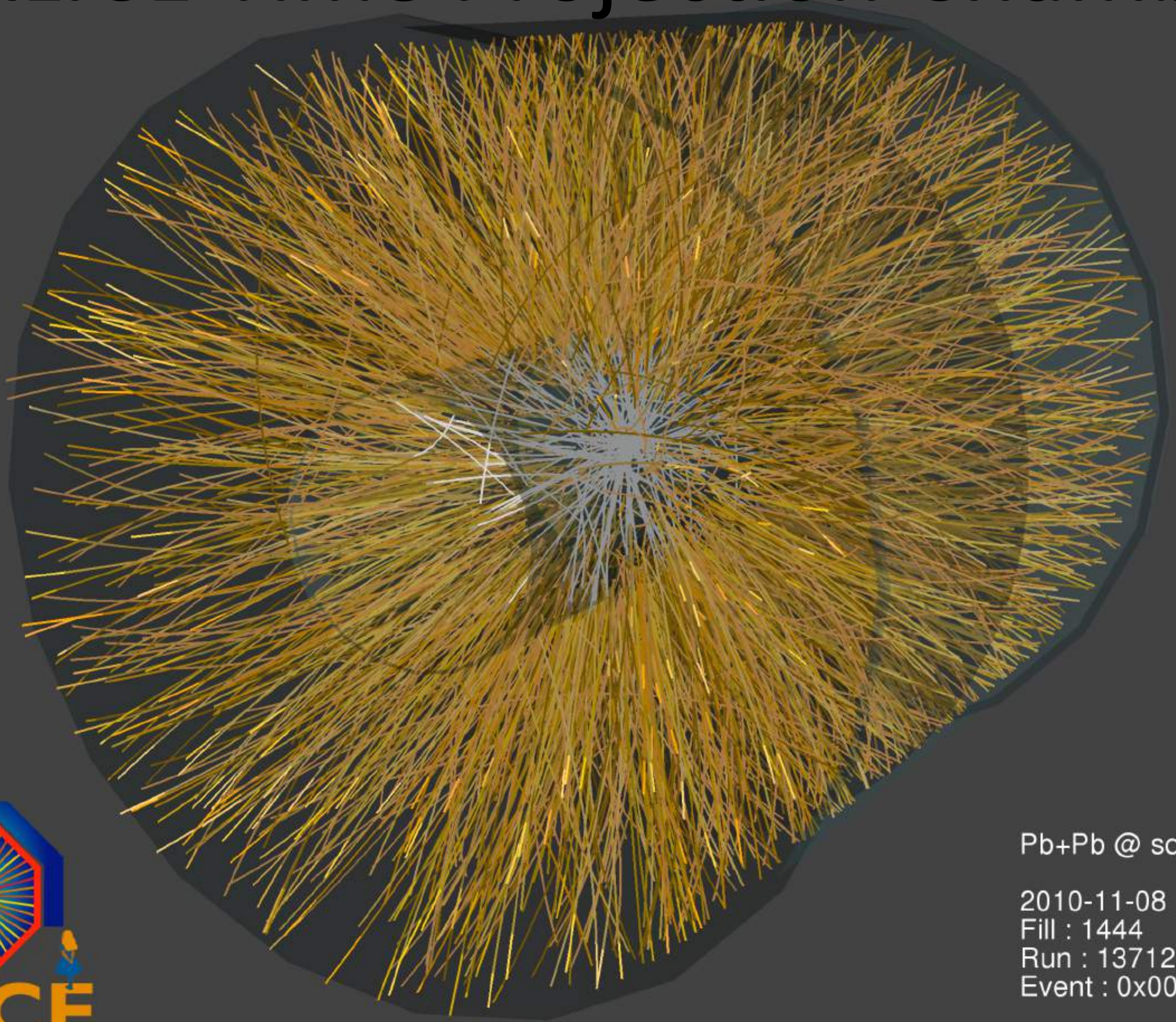
Units: MeV g^{-1} cm^2

MIP looses ~ 13 MeV/cm
[density of copper: 8.94 g/cm^3]

Particle identification with dE/dx measurement



ALICE Time Projection Chamber



Pb+Pb @ sqrt(s) = 2.76 ATeV

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Run : 137124
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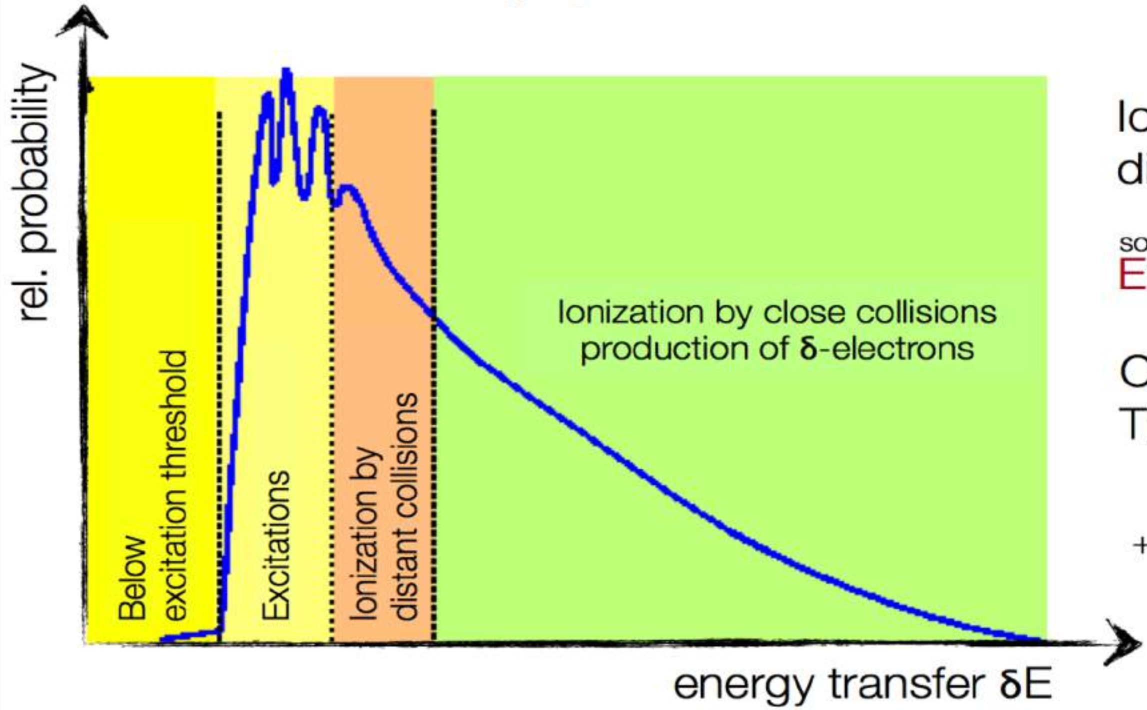
Fluctuation of energy loss

Bethe-Bloch describes mean energy loss; measurement via energy loss **ΔE** in a material of thickness **Δx** with

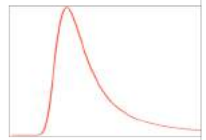
$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N : number of collisions
δE : energy loss in a single collision



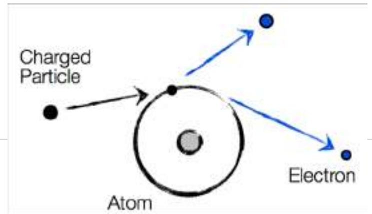
Ionization loss **δE** distributed statistically ...

so-called
Energy loss 'straggling'



Complicated problem ...
Thin absorbers: Landau distribution

Standard Gauss with mean energy loss E_0
\+ tail towards high energies due to **δ**-electrons



$E_\delta > I$

Mean penetration length (range)

Integrate over energy loss
from E down to 0

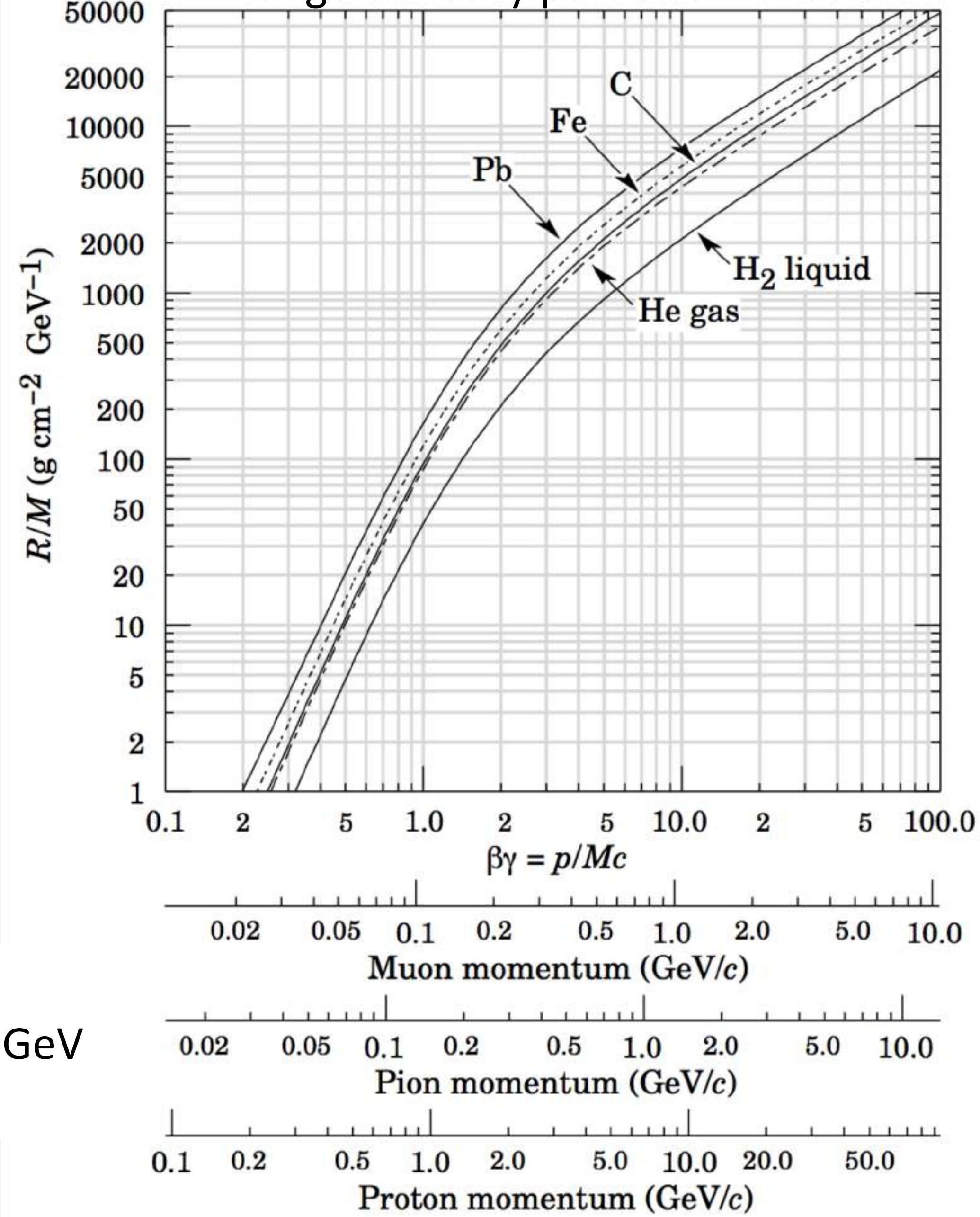
$$R = \int_E^0 \frac{dE}{dE/dx}$$

Example:

Proton with p = 1 GeV
Target: lead with ρ = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}

Range of heavy particles in matter



R ~ (200 g/cm^2/GeV) / (11.34 g/cm^3) * 0.938 GeV
R ~ 20 cm

Ionisation loss of electrons

Reminder for heavy charged particles:

$$\left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$ [·ϱ]

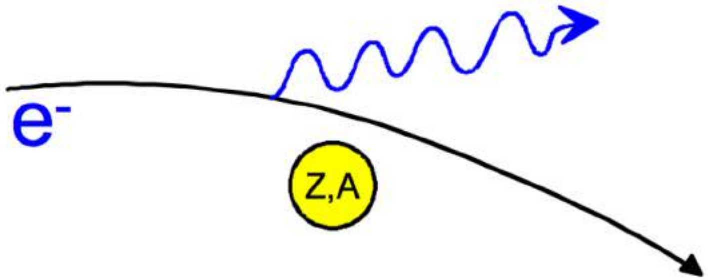
- The Bethe-Bloch formula needs modification as the mass of the incident particle is the same as the mass of atomic electrons
- Scattering of identical (non-distinguishable) particles:

$$\left\langle -\frac{dE}{dx}\right\rangle = \frac{1}{2}K\frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{m_ec^2\beta^2\gamma^2\{m_ec^2(\gamma-1)/2\}}{I^2} + F(\gamma) - \delta\right]$$ [·ϱ]

- Maximal energy loss in the collision: $W_{\max} = m_ec^2(\gamma-1)$
  Non-distinguishable particles: $W_{\max}/2$
- Low energy positrons need different treatment in the calculation, as they are not identical (i.e. they are distinguishable)

Bremsstrahlung

Bremsstrahlung arises if particles
are accelerated in Coulomb field of nucleus



$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to $1/m^2$ ➔ main relevance for electrons ...

... or ultra-relativistic muons

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \quad \text{with} \quad X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm²]

➔ $E = E_0 e^{-x/X_0}$

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy

[i.e. 63%]

X_0: radiation length → important parameter for the design of electromagnetic calorimeters

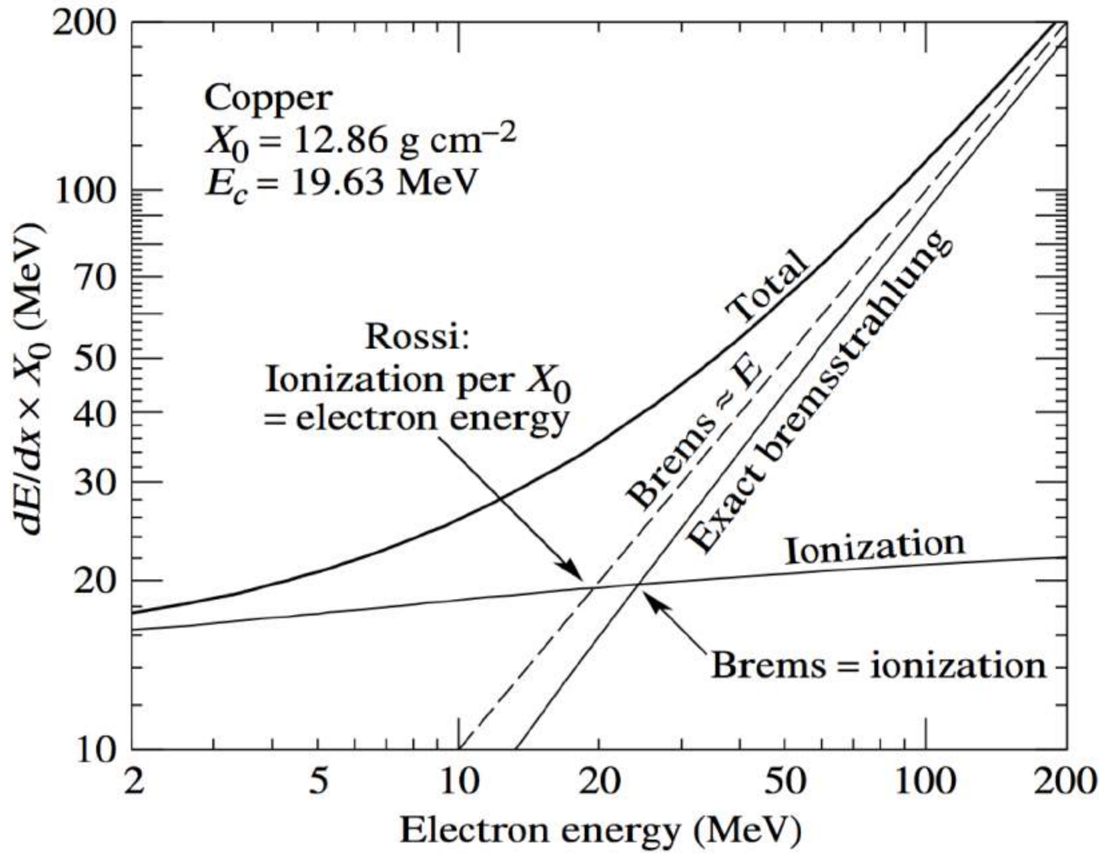
Critical energy

Where ionisation energy loss is equal to radiation energy loss

Critical energy:

$$\frac{dE}{dx}(E_c)\Big|_{\text{Brems}} = \frac{dE}{dx}(E_c)\Big|_{\text{Ion}}$$

$$\left(\frac{dE}{dx}\right)_{\text{Tot}} = \left(\frac{dE}{dx}\right)_{\text{Ion}} + \left(\frac{dE}{dx}\right)_{\text{Brems}}$$



Approximation:

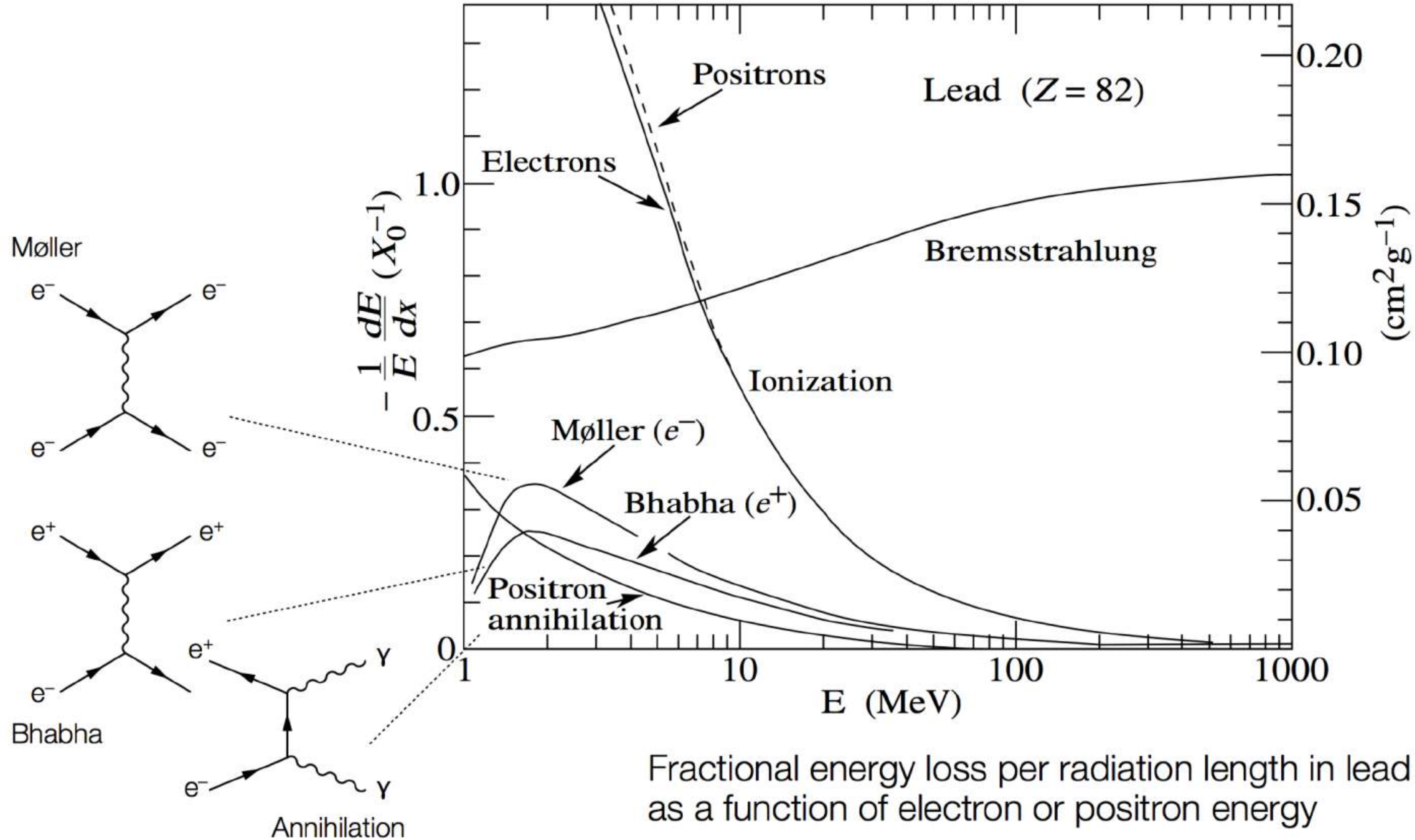
$$E_c^{\text{Gas}} = \frac{710\ \text{MeV}}{Z + 0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610\ \text{MeV}}{Z + 1.24}$$

Example Copper: (Z=29)

$E_c \approx 610/30$ MeV ≈ 20 MeV

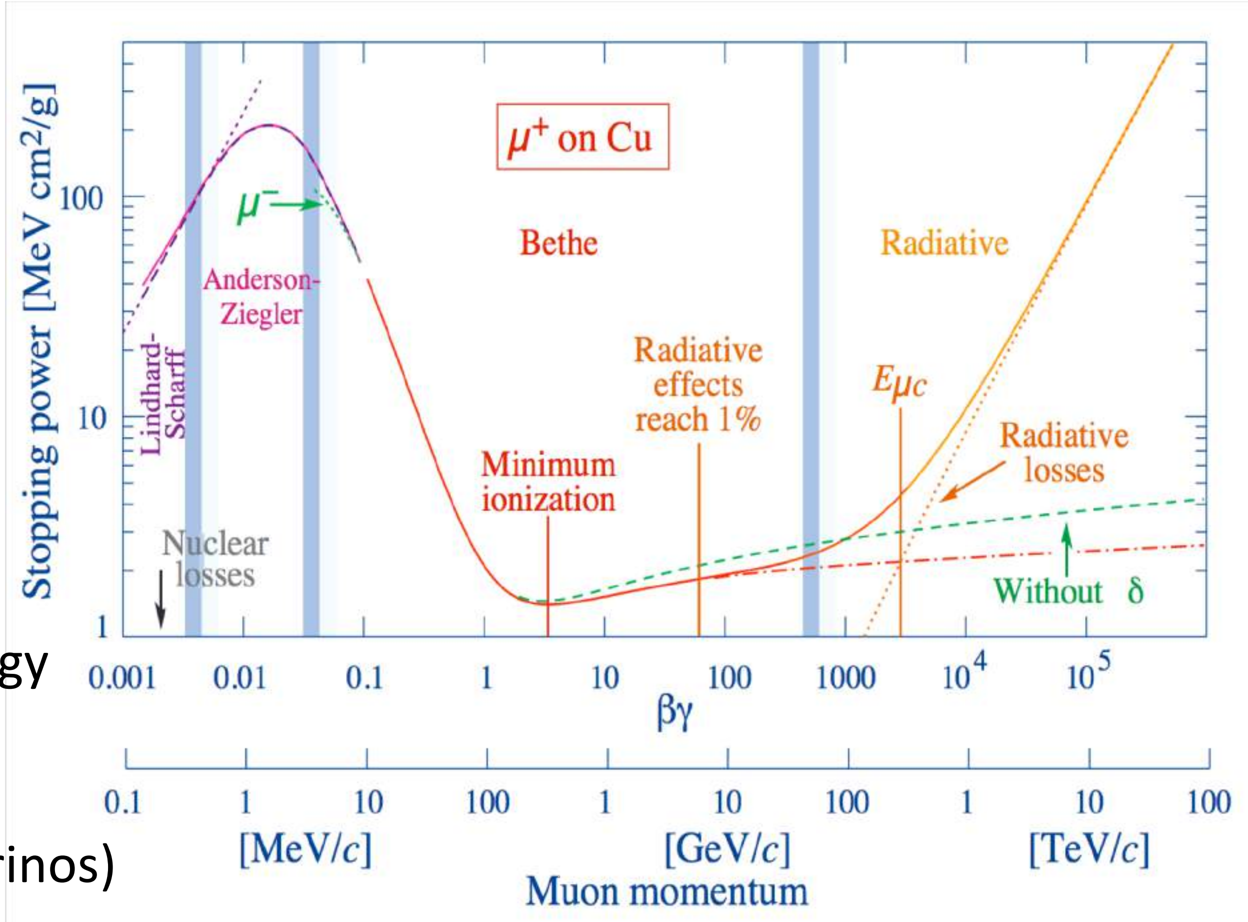
Total energy loss of electrons



Fractional energy loss per radiation length in lead as a function of electron or positron energy

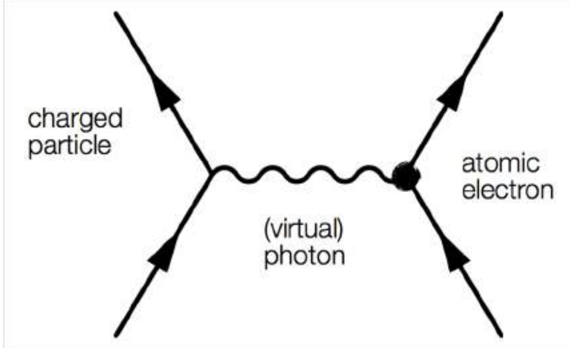
Interactions of muons with matter

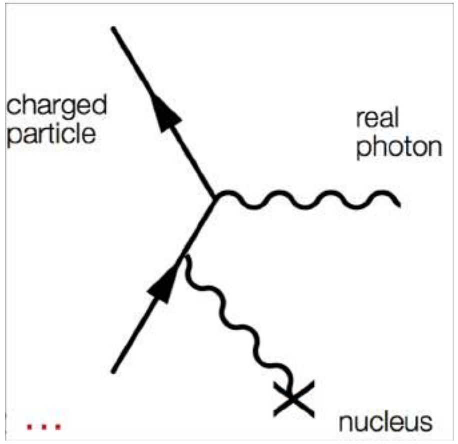
- All charged particles lose energy while traveling through matter via ionisation
- Depending on the particle type, it can lose energy via other processes
- Ionisation loss could be even negligible (see electrons)
- Below 100 GeV energy, ionisation energy loss dominates for muons
- Muons can travel long distances even in dense material (e.g. iron)
  - E=10 GeV muon loses 13 MeV/cm energy in Iron
- At particle accelerators, muons pass through the full detector leaving a long ionisation trail behind → muon identification based on this behaviour as all other particles loose energy via other processes and thus stopped before the outer detector layers (except neutrinos)



Interactions of charged particles

- Charged particles interact electromagnetically via photon exchange with the medium they traverse
- Possible processes
  - Ionisation [see last lecture]
    (short range virtual photons
    ionise the atoms of the medium)
  - Cherenkov radiation
    (if medium transparent, could emit
    EM radiation above a certain
    momentum)
  - Transition radiation
    (EM radiation if dielectric constant changes
    at the boundary of two different media)
  - Bremsstrahlung [see before]
    (Particle decelerating in Coulomb field emits
    a real photon)

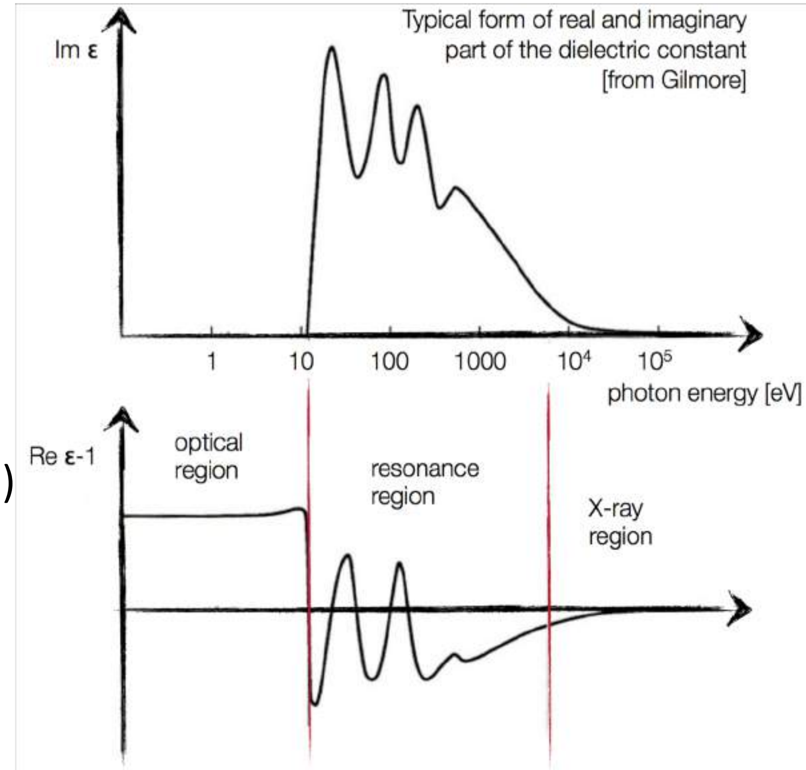




- To calculate energy loss or intensity of emitted radiation, one needs to consider
  - Speed of charged particle: v= β·c
  - Dielectric constant of medium: $\varepsilon = \varepsilon_1 + i\varepsilon_2$
    - Describes the interactions of (virtual) photons with the atoms of the medium
    - ε_1 : refraction (→ changes direction of wave propagation)

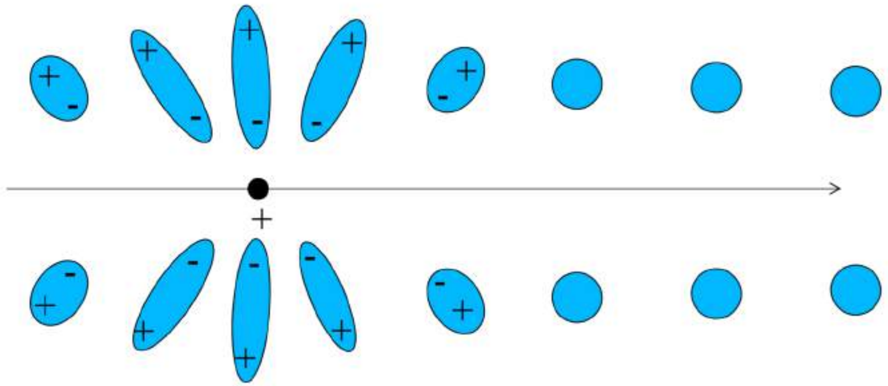
$$u(\omega) = \frac{c}{\sqrt{\epsilon(\omega)}}$$

    - ε_2 : photon absorption
      (→ absorption cross-section)

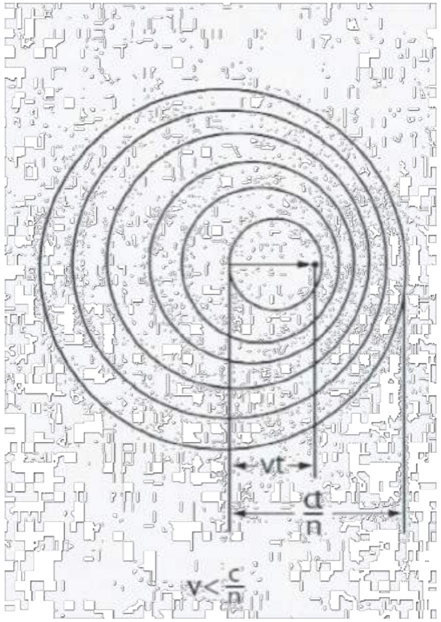


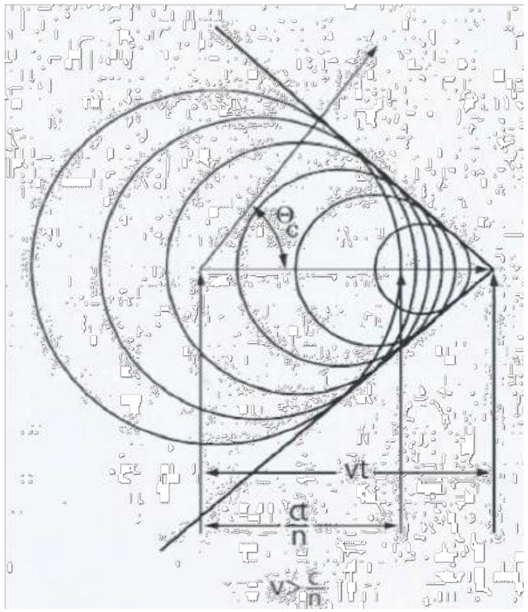
Cherenkov radiation

- Dielectric medium electrons polarized by a moving charged particle.



- De-exitation give rise to a coherent radiation.
- When a charged particle moves faster than the phase speed of light in a medium, electrons interacting with the particle can emit coherent photons while conserving energy and momentum.
- Emission is coherent because in phase with the particle velocity.
- Cerenkov radiation consist of a shock wave
- Similar to Doppler effect or Mach shock waves
- Pavel A. Čerenkov and Vavilov discovered the radiation in 1934, Igor Tamm and Ilya Frank explained it in 1937.





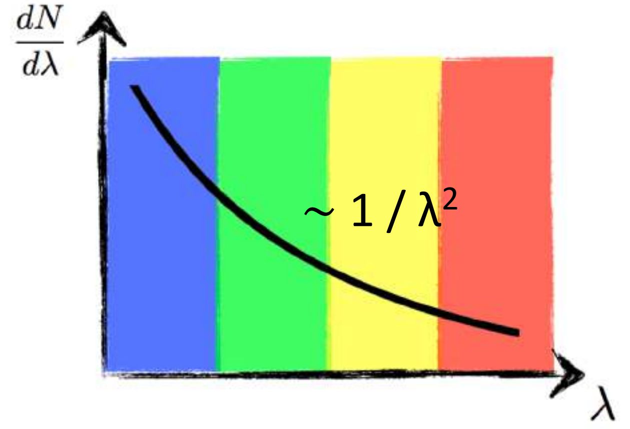
Cherenkov radiation

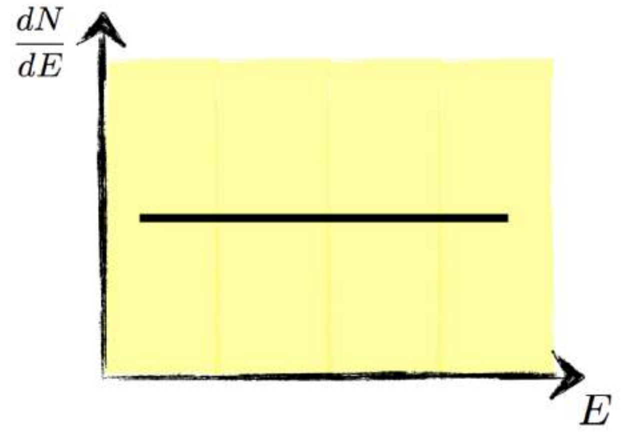
- Low energy photons (E < excitation energy):
  $\varepsilon_2 = 0 \rightarrow \sigma_{\gamma\ \text{absorbtion}} = 0$

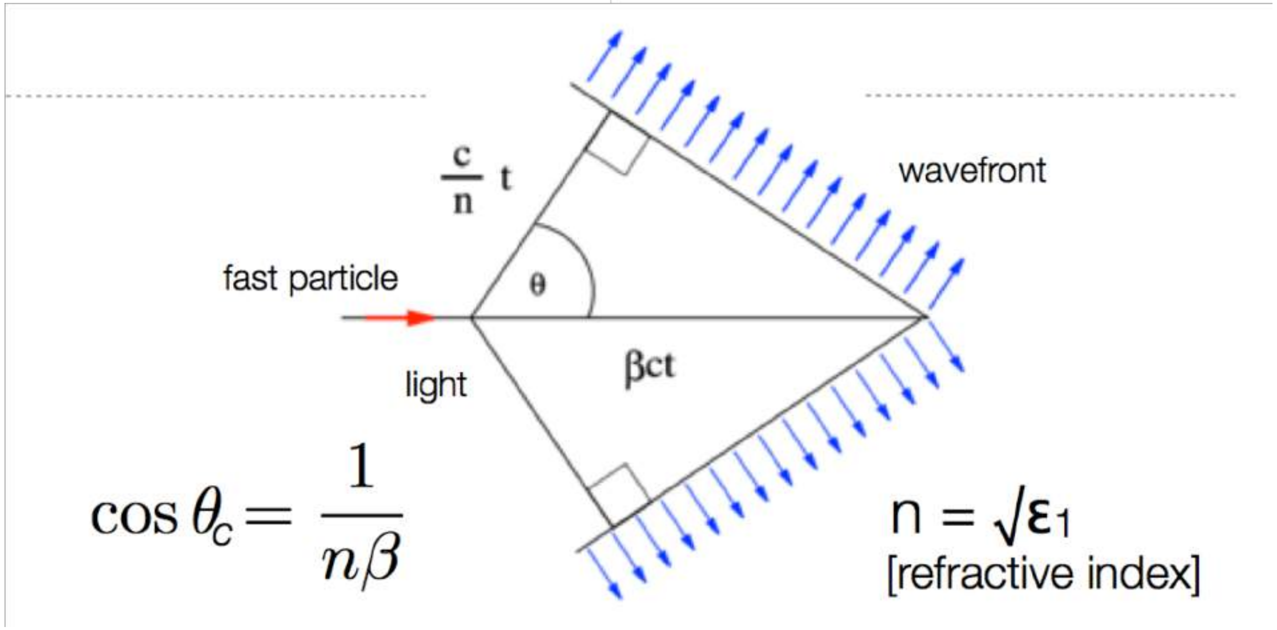
$$\frac{d\sigma}{dE} = \frac{z^2\alpha}{\beta^2\pi}\frac{1}{N_\alpha \hbar c}\left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2}\right)\Theta$$

$$\Theta = \arg(1-\varepsilon_1\beta^2 + i\varepsilon_2\beta^2) = \arg(1-\varepsilon_1\beta^2)$$

- Threshold behaviour at $\varepsilon_1 > 1/\beta^2$ value: $0 \rightarrow \pi$
- Cherenkov threshold: $1 < \beta\sqrt{\varepsilon_1} = \beta \cdot n$, thus $\beta > 1/n$
- Cherenkov angle: $\cos\theta_c = 1/(n \cdot \beta)$







Cherenkov radiators

Parameters of Typical Radiator

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Air	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Water	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

Note: Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%).

Example: $E_k = E - E_0 = (\gamma - 1) m_0 c^2$

[Proton with E_{kin} =1 GeV passing through 1 cm water]

$\beta = p/E \approx 0.875$; $\cos\theta_C = 1/n\beta = 0.859$ ➛ $\theta_C = 30.8°$

$d^2N/(dEdx) = 370 \sin^2\theta_C$ eV^{-1} $cm^{-1} \approx 100$ eV^{-1} cm^{-1}

➛ ΔE_{loss} = $\langle E \rangle d^2N/(dEdx)\, \Delta E\, \Delta x$
= 2.5 eV · 100 eV^{-1} cm^{-1} · 5 eV · 1 cm = 1.25 keV

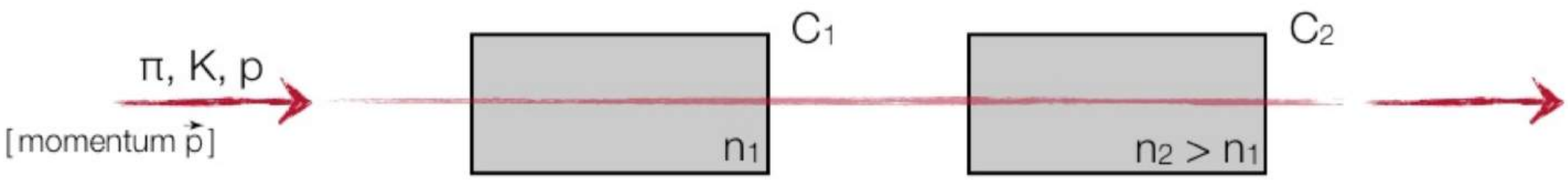
Visible light only!
[E = 1 - 5 eV; λ = 300 - 600 nm]

ΔE_{loss} < 1.25 keV

Particle identification using Cherenkov radiation

Threshold detection:

Observation of Cherenkov radiation → $\beta > \beta_{thr}$



Choose n_1, n_2 in such a way that for:

n_2 : $\beta_\pi, \beta_K > 1/n_2$ and $\beta_p < 1/n_2$

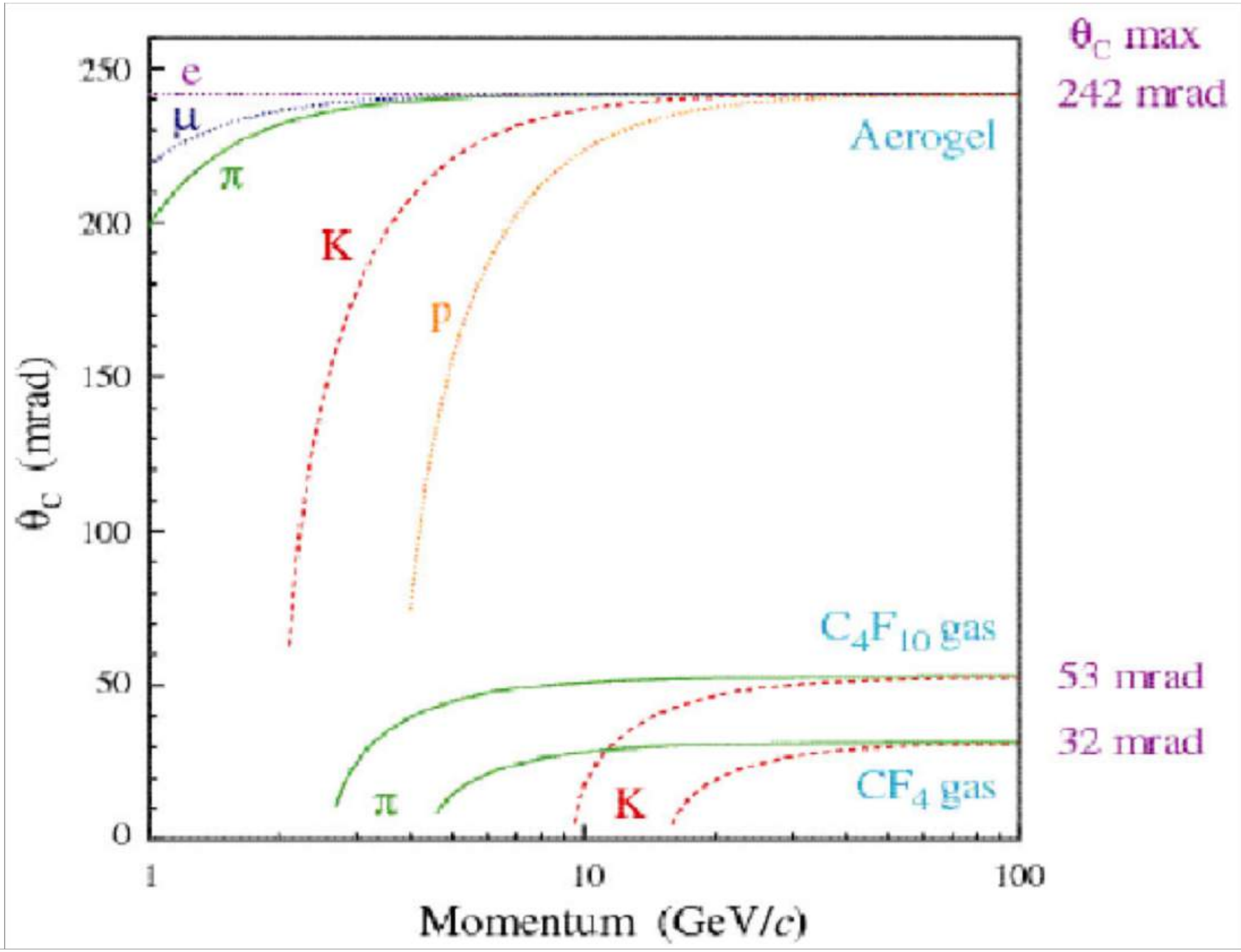
n_1 : $\beta_\pi > 1/n_1$ and $\beta_K, \beta_p < 1/n_1$

Light in C_1 and C_2 → identified pion

Light in C_2 and not in C_1 → identified kaon

Light neither in C_1 and C_2 → identified proton

Particle identification using Cherenkov radiation



Cherenkov angle
Number of photons

grows with β and reaches asymptotic value for $\beta = 1$

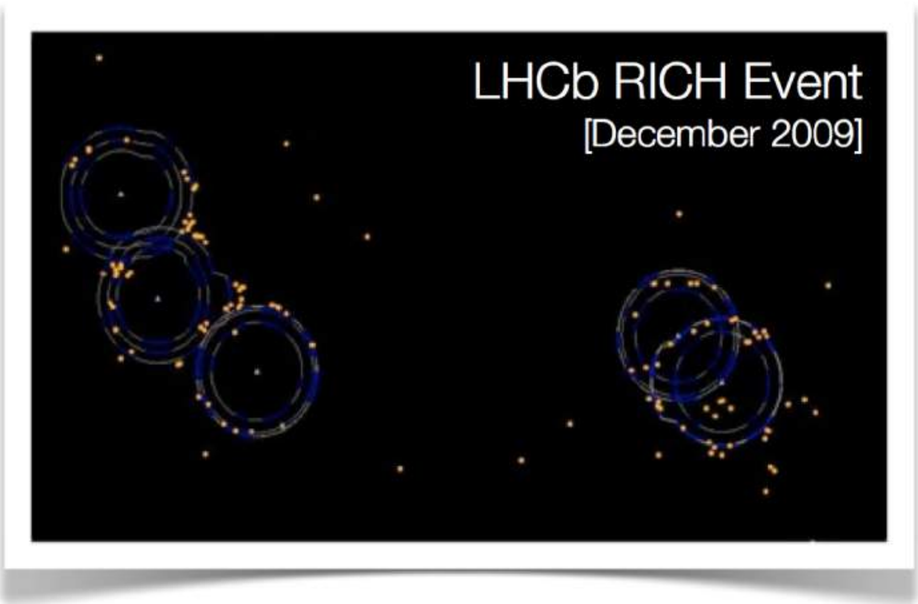
[$\theta_{max} = \arccos(1/n)$; $N_\infty = x \cdot 370/\text{cm}\,(1-1/n^2)$]

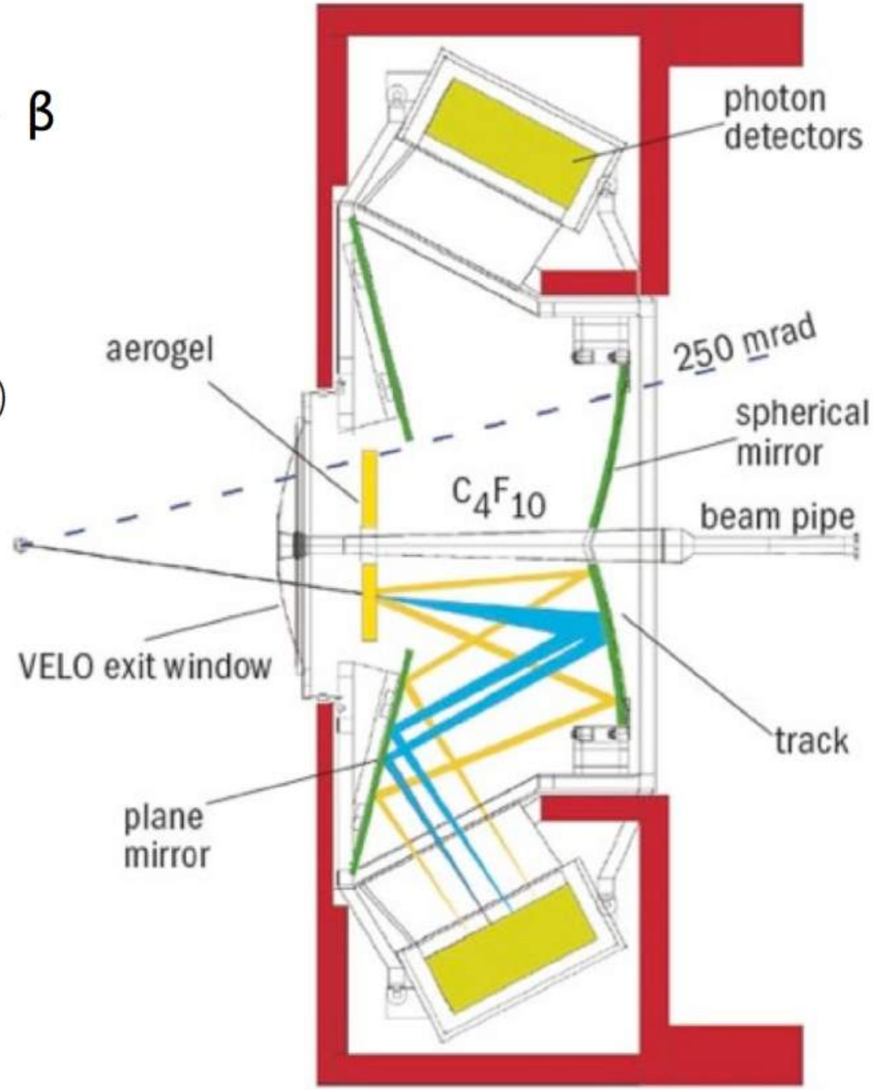
Particle identification using Cherenkov radiation

Measurement of Cherenkov angle:
Use medium with known refractive index n ➛ β

Principle of:

RICH (Ring Imaging Cherenkov Counter)
DIRC (Detection of Internally Reflected Cherenkov Light)
DISC (special DIRC; e.g. Panda)





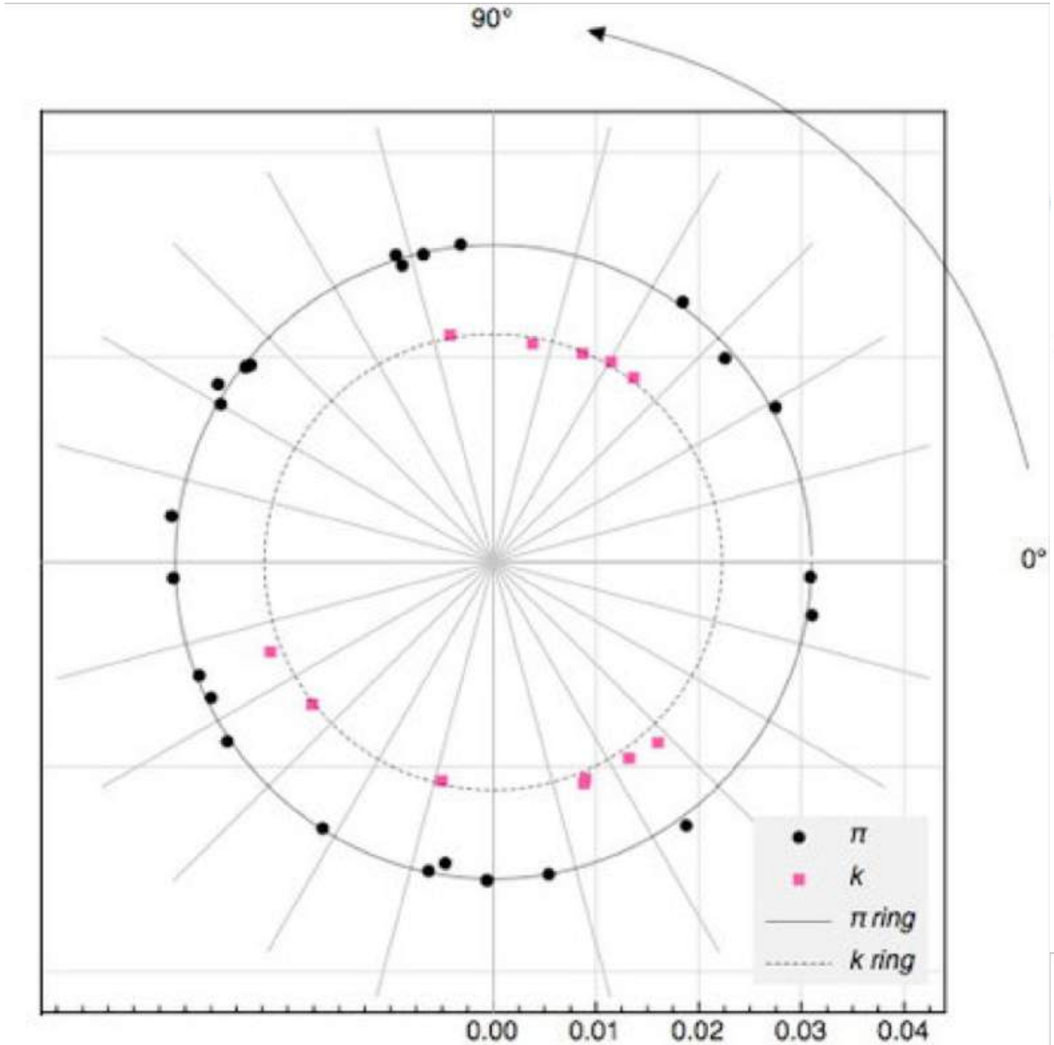
LHCb RICH

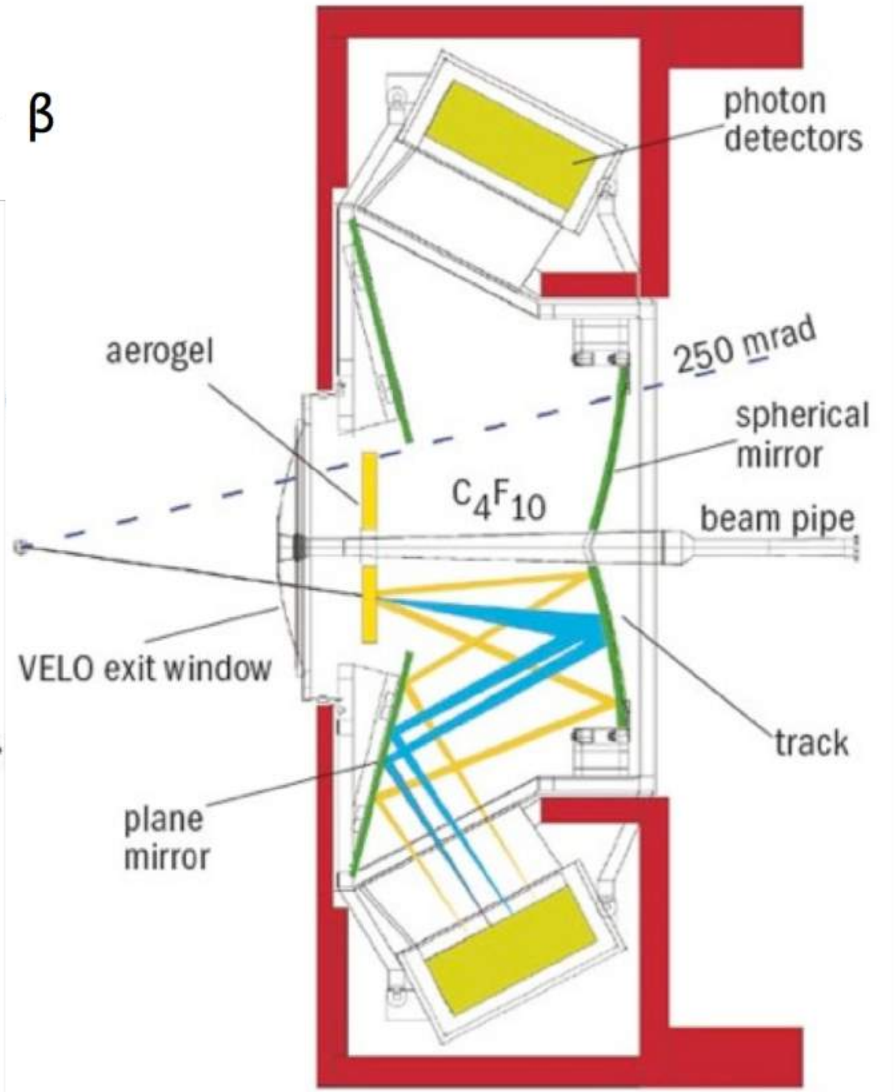
$$\cos\theta_C = \frac{1}{n\beta}$$

Particle identification using Cherenkov radiation

Measurement of Cherenkov angle:

Use medium with known refractive index n ➛ β





LHCb RICH

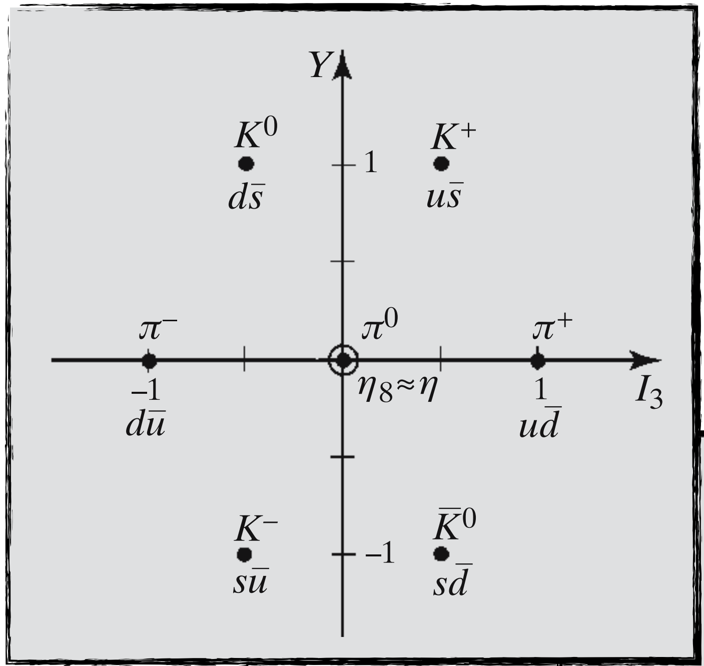
$$\cos\theta_c = \frac{1}{n\beta}$$

Homework #4

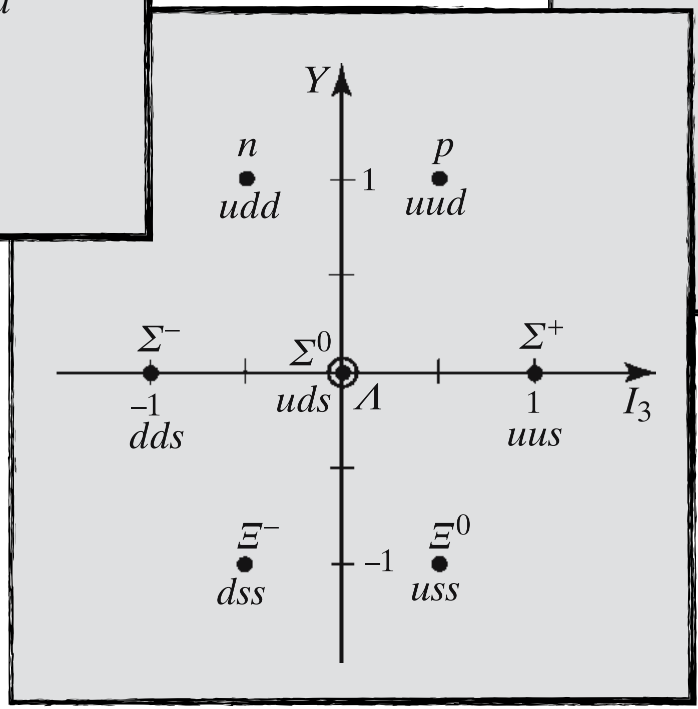
- Calculate the distance travelled by a 15 GeV charged pion, a 15 GeV J/psi and a 15 GeV charged B meson before decaying. How can we use this information to identify them?
- We want to separate pions, kaons and protons of 1 GeV/c momenta using Cherenkov counters. What materials shall we use to build a suitable detector?
- A proton moving in water emits Cherenkov radiation in a cone making an angle of 40 degree with the electron's direction of motion. Compute the kinetic energy of the proton. How many photons are emitted and how much energy is lost by the proton via Cherenkov radiation per centimeter? How does this compare to the energy lost via ionization by the same proton per centimeter?
- A 1 MeV proton loses 5 keV energy by ionisation in a given detector. How much energy is lost by a 6 MeV ^{12}C nuclei in the same detector? And if it has only 1 MeV energy?

Extra: Alapvető ismeretek

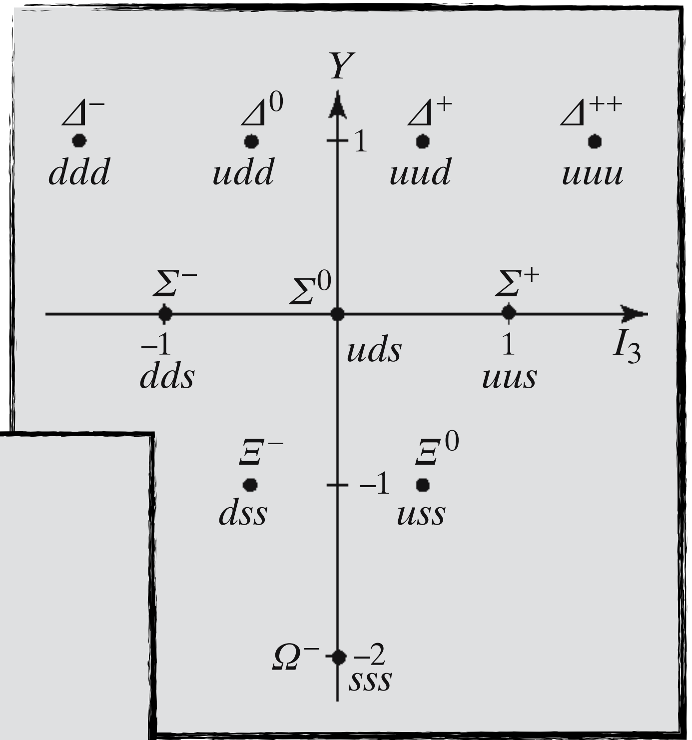
Baryons and Mesons



Meson octet



Baryon octet

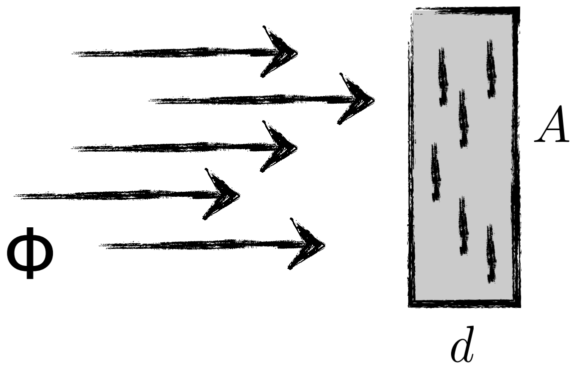


Baryon decuplet

Cross Section – Definition

Incoming flux:

$$\Phi = \frac{1}{A} \cdot \frac{\Delta N}{\Delta t} = \frac{\dot{N}_{\mathrm{in}}}{A}$$



Reaction rate:

$$\dot{N}_{\mathrm{reac}} = \dot{N}_{\mathrm{in}} \frac{A_{\mathrm{tar}}}{A} = \Phi \cdot A_{\mathrm{tar}}$$
$$= \Phi \cdot N_{\mathrm{tar}} \cdot \sigma$$

Absorbing target area

Effective cross section

$$A_{\mathrm{tar}} = \sigma \cdot N_{\mathrm{tar}}$$
$$= \sigma \cdot \frac{\rho \cdot Ad}{m_{\mathrm{mol}}} \cdot N_{\mathrm{A}}$$

with

ρ : target density

m_{mol} : molar mass

N_{A} : $6.022 \cdot 10^{23}\ \mathrm{mol}^{-1}$

Cross section:

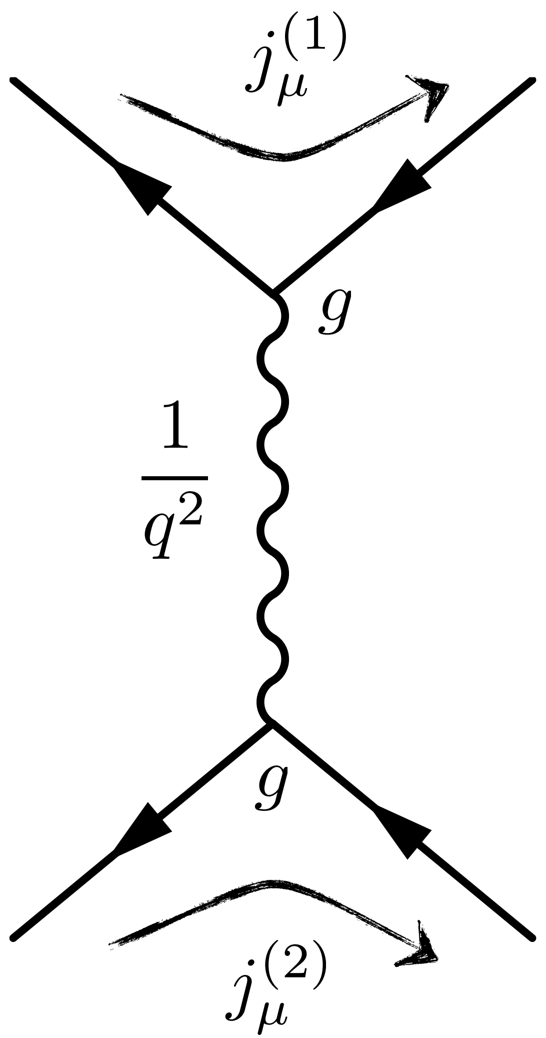
$$\sigma = \frac{\text{Number of reactions (of given type) per unit time}}{\text{Incoming flux} \cdot \text{Number of target particles}}$$

$$= \frac{\dot{N}_{\mathrm{reac}}}{\Phi \cdot N_{\mathrm{tar}}} = \frac{\dot{N}_{\mathrm{reac}}}{\dot{N}_{\mathrm{in}} \cdot N_{\mathrm{A}} \cdot \rho \cdot d / m_{\mathrm{mol}}}$$

Transition rate W_{fi}

Unit: $[\sigma] = \mathrm{cm}^2$

Cross Section – Using Feynman Diagrams



Fermi's Golden Rule

$$W_{\text{fi}} = 2\pi \, |M_{\text{fi}}|^2 \cdot \frac{dN}{dE_{\text{f}}}$$

Transition probability

Matrix element

Phase space

4-vector current

$$M_{\text{fi}} = -i \int j_\mu^{(1)} \cdot \left(\frac{1}{q^2}\right) \cdot j_\mu^{(2)} d^4x$$

Propagator

$$\sigma \sim |M_{\text{fi}}|^2$$

$$\sim g^4 \cdot \left(\frac{1}{q^4}\right)$$

Measuring Particles

Particles are characterized by

Mass	[Unit: eV/c² or eV]
Momentum	[Unit: eV/c or eV]
Energy	[Unit: eV]
Charge	[Unit: e]

[+ Spin, Lifetime ...]

$eV = 1.6 \cdot 10^{-19}$ J
$c = 299\,792\,458$ m/s
$e = 1.602176487(40) \cdot 10^{-19}$ C

Relativistic kinematics:

$$E^2 = \vec{p}^{\,2}c^2 + m^2c^4$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E = m\gamma c^2 = mc^2 + E_{\mathrm{kin}} \qquad \vec{p} = m\gamma\vec{\beta}c \qquad \vec{\beta} = \frac{\vec{p}c}{E}$$

Particle Identification via measurement of

e.g. $(E, \vec{p}, Q)$ or $(\vec{p}, \beta, Q)$
$(\vec{p}, m, Q)$ ...

HEP and SI Units

Quantity	HEP units	SI Units
length	1 fm	10^{-15} m
energy	1 GeV	$1.602\cdot 10^{-10}$ J
mass	1 GeV/c^2	$1.78\cdot 10^{-27}$ kg
$\hbar$=h/2	$6.588\cdot 10^{-25}$ GeV s	$1.055\cdot 10^{-34}$ Js
c	$2.988\cdot 10^{23}$ fm/s	$2.988\cdot 10^{8}$ m/s
$\hbar c$	0.1973 GeV fm	$3.162\cdot 10^{-26}$ Jm

Natural units ($\hbar$ = c = 1)	
mass	1 GeV
length	1 GeV^{-1} = 0.1973 fm
time	1 GeV^{-1} = $6.59\cdot 10^{-25}$ s

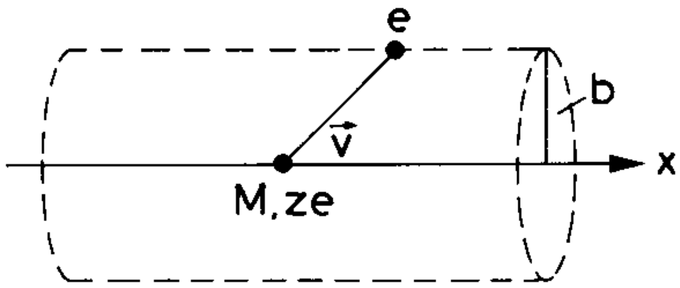
Extra: EM kölcsönhatások

Bethe-Bloch – Classical Derivation

Bohr 1913

Particle with charge ze and velocity v moves through a medium with electron density n.

Electrons considered free and initially at rest.



Interaction of a heavy charged particle with an electron of an atom inside medium.

Momentum transfer:

Symmetry!

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v} \qquad \Delta p_{||} \ : \ \text{averages to zero}$$

$$= \int_{-\infty}^{\infty} \frac{ze^2}{(x^2+b^2)} \cdot \frac{b}{\sqrt{x^2+b^2}} \cdot \frac{1}{v}\, dx = \frac{ze^2 b}{v} \left[\frac{x}{b^2\sqrt{x^2+b^2}} \right]_{-\infty}^{\infty} = \frac{2ze^2}{bv}$$

More elegant with Gauss law:
[infinite cylinder; electron in center]

$$\int E_\perp \,(2\pi b)\, dx = 4\pi (ze) \rightarrow \int E_\perp dx = \frac{2ze}{b}$$

and then ...

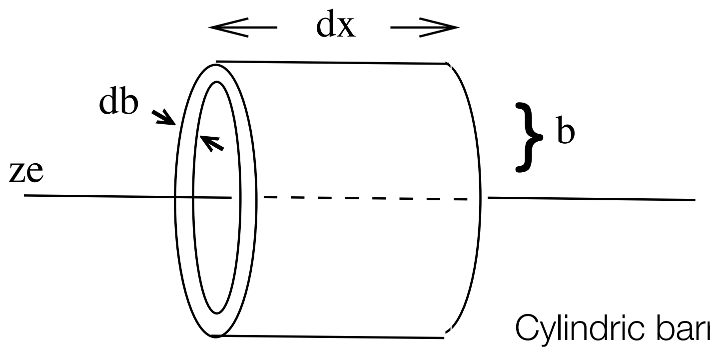
$$\begin{cases} F_\perp = eE_\perp \\ \Delta p_\perp = e \int E_\perp \frac{dx}{v} = \frac{2ze^2}{bv} \end{cases}$$

Bethe-Bloch – Classical Derivation

Bohr 1913

Energy transfer onto single electron
for impact parameter b:

$$\Delta E(b) = \frac{\Delta p^2}{2m_e}$$



Cylindric barrel
with N_e electrons

Consider cylindric barrel ➛ $N_e = n \cdot (2\pi b) \cdot db\,dx$

Energy loss per path length dx for
distance between b and b+db in medium with electron density n:

Energy loss!

$$-dE(b) = \frac{\Delta p^2}{2m_e} \cdot 2\pi nb\,db\,dx = \frac{4z^2e^4}{2b^2v^2m_e} \cdot 2\pi nb\,db\,dx = \frac{4\pi\,n\,z^2e^4}{m_e v^2}\frac{db}{b}dx$$

Diverges for b ➛ 0; integration only
for relevant range [b_{min}, b_{max}]:

Bohr 1913

$$-\frac{dE}{dx} = \frac{4\pi\,n\,z^2e^4}{m_e v^2} \cdot \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi\,n\,z^2e^4}{m_e v^2} \ln\frac{b_{max}}{b_{min}}$$

Bethe-Bloch – Classical Derivation

Bohr 1913

Determination of relevant range [b_{min}, b_{max}]:
[Arguments: $b_{min} > \lambda_e$, i.e. de Broglie wavelength; $b_{max} < \infty$ due to screening ...]

$$b_{\rm min} = \lambda_{\rm e} = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_{\rm e} v}$$

Use Heisenberg uncertainty principle or that electron is located within de Broglie wavelength ...

$$b_{\rm max} = \frac{\gamma v}{\langle \nu_{\rm e} \rangle} \; ; \quad \left[\; \gamma = \frac{1}{\sqrt{1-\beta^2}} \; \right]$$

Interaction time (b/v) must be much shorter than period of the electron (γ/ν_e) to guarantee relevant energy transfer ...

[adiabatic invariance]

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_{\rm e}\, c^2 \beta^2}\, n \cdot \ln \frac{m_{\rm e}\, c^2 \beta^2 \gamma^2}{2\pi\hbar\, \langle \nu_{\rm e} \rangle}$$

Deviates by factor 2 from QM derivation

Electron density: $n = N_A \cdot \rho \cdot Z/A$!!

Effective Ionization potential: $I \sim h\langle \nu_e \rangle$

Interaction Cross Section

[Allison, Cobb; Ann. Rev. Nucl. Part. Sci. 30 (1980) 253]

Photo-absorption cross section

$$\frac{d\sigma}{dE} = \frac{z^2\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left[(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2\right]^{-\frac{1}{2}}$$

Relativistic rise; polarization effect; saturation ...

$$+ \frac{z^2\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left(\frac{2mc^2\beta^2}{E}\right)$$

Rutherford scattering; photoelectric emission

$$+ \frac{z^2\alpha}{\beta^2\pi}\frac{1}{E^2}\int_0^E\frac{\sigma_\gamma(E')}{Z}dE'$$

Compton scattering; Prod. of δ-electrons

$$+ \frac{z^2\alpha}{\beta^2\pi}\frac{1}{N_\alpha\hbar c}\left(\beta^2-\frac{\epsilon_1}{|\epsilon|^2}\right)\Theta$$

Cherenkov & Transition Radiation

Yields energy loss dE/dx

Particle charge

Density of medium

Phase of $1-\epsilon_1\beta^2 + i\epsilon_2\beta^2$

Bethe Bloch from dσ/dE

Integrate cross section over all γ-energies:

$$\frac{dE}{dx} = \int_0^\infty EN \frac{d\sigma}{dE} dE$$

Electron density within medium (→ N)

Energy of exchanged photon; $E = \hbar\omega$ (→ dE)

$$\frac{d\sigma}{dE} = \frac{z^2\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left[(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2\right]^{-\frac{1}{2}} \quad (1)$$

$$(2) \quad + \frac{z^2\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left(\frac{2mc^2\beta^2}{E}\right)$$

$$+ \frac{z^2\alpha}{\beta^2\pi}\frac{1}{E^2}\int_0^E \frac{\sigma_\gamma(E')}{Z}dE' \quad (3)$$

$$+ \ \ldots$$

Bethe-Bloch results as an approximation:

$$\frac{dE}{dx} \propto \frac{1}{\beta^2}\cdot\ln\frac{2mc^2\beta^2\gamma^2T_{\max}}{I^2}$$

(2) → $2mc^2\beta^2$; (1), (3) → $\gamma^2 T_{\max}$

Cherenkov Radiation

4

For photon energies below the excitation energy:

$$\frac{d\sigma}{dE} = \frac{z^2\alpha}{\beta^2\pi}\frac{1}{N_\alpha \hbar c}\left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2}\right)\Theta$$

$\varepsilon_2 = 0$ and $\sigma_\gamma = 0$ ➞ only last term of $d\sigma/dE$ contributes ...

Threshold behavior via phase Θ:

$\Theta = \arg(1-\varepsilon_1\beta^2 + i\varepsilon_2\beta^2) = \arg(1-\varepsilon_1\beta^2)$

Jumps from 0 to π for: $\varepsilon_1 > 1/\beta^2$ or

$1 < v/c\,\sqrt{\varepsilon_1}$ ➞ Cherenkov threshold.

➞ $$\sqrt{\epsilon}\frac{v}{c}\cos\theta_C = 1$$

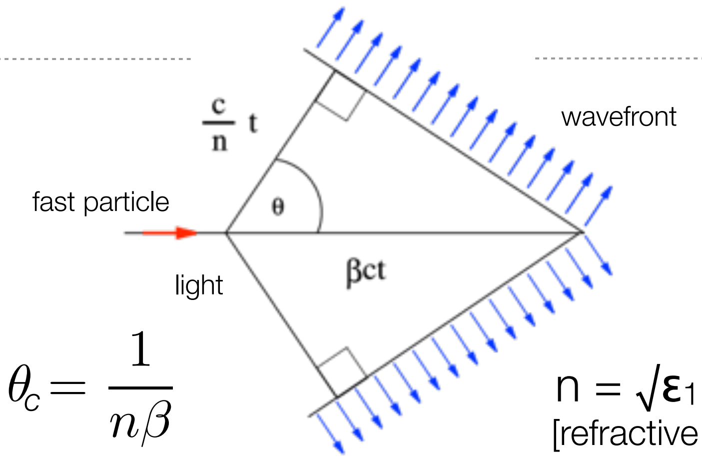
[θ_c: Cherenkov angle]

Kinematics: $\omega = \vec{v}\vec{k} = vk\cos\theta_c$

[from $p' = p - p_\gamma$ assuming $\hbar\omega \ll \gamma M c^2$]

Dispersion relation: $\omega^2 = k^2c^2/\varepsilon_1$

➞ $\varepsilon_1 \cdot v^2/c^2 \cos\theta_c = 1$



$$\cos\theta_c = \frac{1}{n\beta}$$

$n = \sqrt{\varepsilon_1}$
[refractive index]

Cherenkov Radiation – Properties

Number of emitted photons
per unit length:

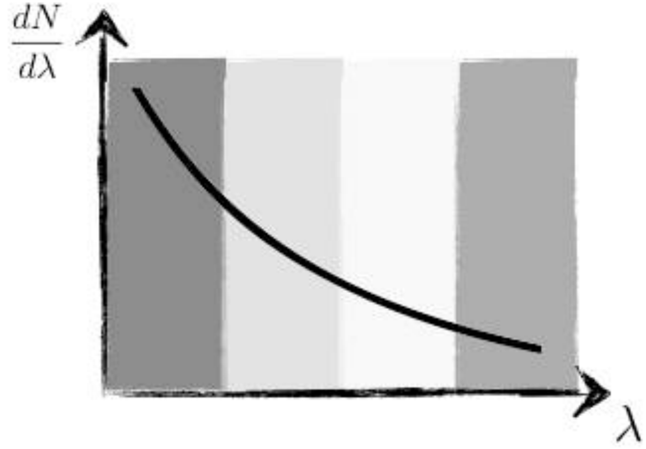
from (4)

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_C$$

Integrate over sensitivity range:
[for typical Photomultiplier]

$$\frac{dN}{dx} = \int_{350\ \mathrm{nm}}^{550\ \mathrm{nm}} d\lambda \frac{d^2N}{d\lambda dx}$$

$$= 475\ z^2 \sin^2\theta_C\ \mathrm{photons/cm}$$



$$\frac{d^2N}{dEdx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin^2\theta_C$$

$$\approx \mathrm{const}$$

For single charged
particle:

$$\frac{d^2N}{dEdx} = 370\ \sin^2\theta_C\ \mathrm{eV^{-1}\ cm^{-1}}$$